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RESEARCH PAPER

A network equilibrium analysis on destination, route and parking choices with mixed gasoline and electric vehicular flows

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Abstract In many countries across the world, fossil fuels, especially petroleum, are the largest energy source for powering the socio-economic system, and the transportation sector dominates the consumption of petroleum in these societies. As the petroleum price continuously climbs and the threat of global climate changes becomes more evident, the world is now facing critical challenges in reducing petroleum consumption and exploiting alternative energy sources. A massive adoption of plug-in electric vehicles (PEVs), especially battery electric vehicles (BEVs), offers a very promising approach to changing the current energy consumption structure and diminishing greenhouse gas emissions and other pollutants. Understanding how individual electric vehicle drivers behave subject to the technological restrictions and infrastructure availability and estimating the resulting aggregate supply–demand effects on urban transportation systems is not only critical to transportation infrastructure development, but also has determinant implications in environmental and energy policy enactment. This paper presents an

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equilibrium-based analytical tool for quantifying travel choice patterns in urban transportation networks with both gasoline and electric vehicular flows. Specifically, a network equilibrium problem with combined destination, route and parking choices subject to the driving range limit and alternative travel cost composition associated with BEVs are formulated, solved, and numerically analyzed under different network settings and scenarios. The defined problem introduces a new dimension of modeling network equilibrium problems with side constraints. The practical significance of the developed tool lies in its solution tractability and extension capability and its ease of being embedded into the existing urban travel demand forecasting framework.

Keywords Electric vehicles · Network equilibrium · Travel demand analysis · Travel choices · Distance constraint · Partial linearization method

List of symbols

Variables

- $f_{k,g}^{rs,n}$ trip flow rate of GVs traveling on path k between O-D pair r-s and entering the ordinary parking facility at destination s
- $f_{k,\mathrm{e}}^{rs,n}$ trip flow rate of BEVs traveling on path k between O-D pair r-s and entering the ordinary parking facility at destination s
- $f_{k,\mathrm{e}}^{rs,e}$ trip flow rate of BEVs traveling on path k between O-D pair r-s and entering the special parking facility at destination s
- $x_{a,\mathrm{g}}$ traffic flow rate of GVs on link a
- $x_{a,\mathrm{e}}$ traffic flow rate of BEVs on link a
- x_a total traffic flow rate on link a
- q_{g}^{rs} trip flow rate of GVs between O-D pair r-s
- q_{e}^{rs} trip flow rate of BEVs between O-D pair r-s
- D_{g}^{s} arrival trip flow rate of GVs at destination s
- D_{e}^{s} arrival trip flow rate of BEVs at destination s
- $R^{s,n}$ arrival trip flow rate at the ordinary parking facility at destination s
- $R^{s,e}$ arrival trip flow rate at the special parking facility at destination s
- P_{g}^{rs} probability of GVs departing from origin r choosing destination s
- P_{e}^{rs} probability of BEVs departing from origin r choosing destination s
- $t^{s,n}$ parking access and search time of the ordinary parking facility at destination s
- $t^{s,e}$ parking access and search time of the special parking facility at destination s
- t_a travel time of link a
- $p_{k,\mathrm{g}}^{rs}$ generalized travel cost of GVs along path k between O-D pair r-s
- $p_{k,\mathrm{e}}^{rs}$ generalized travel cost of BEVs along path k between O-D pair r-s

Parameters

- O_{g}^{r} departing trip flow rate of GVs from origin r
- O_{e}^{r} departing trip flow rate of BEVs from origin r
- $\delta_{a,k}^{rs}$ 1 if link a is on path k from r to s, and 0 otherwise
- ρ value of time
- u_{g} unit operating cost of GVs on roadway links
- u_{e} unit operating cost of BEVs on roadway links

$c^{s,n}$ parking fee of the ordinary parking facility at destination s
$c^{s,e}$ parking fee of the special parking facility at destination s
d_a physical length of link a
l_k^{rs} physical length of path k between O-D pair r-s
D driving range limit of BEVs
γ_g, γ_e scale parameters of the multinomial logit models of GVs and BEVs for destination choice, respectively

Introduction

The human society's overdependence on petroleum has contributed to a suite of serious economic, security, geopolitical, and environmental problems. Reducing petroleum use and seeking energy diversity have become a growing global interest and public consensus in many countries. In the US, the transportation sector accounts for 27 % of its greenhouse gas emissions and 70 % of the petroleum consumption (EIA 2010). The majority of the petroleum consumption in the transportation sector is for fueling light-duty vehicles. Encouraging viable energy alternatives to power vehicles and hence changing the current energy consumption structure are becoming an urgent task for enhancing the nation's energy security and economic and environmental sustainability.

With innovations in battery technologies and expectedly rapid expansions of electricity-charging infrastructures, powering vehicles with electricity is of increasing interest in the public and provides a promising strategy in the search for ways to reduce petroleum use. In the US, the federal government recently announced its strategic plan of promoting electricity as a replacement alternative to petroleum for transportation purposes. An official domestic goal of putting one million plug-in electric vehicles on the road by 2015 has been established, and a variety of fiscal and institutional policies proposed and implemented by the federal, state, and local governments to encourage the electrification of transportation systems (Becker et al. 2009).

Different from gasoline vehicles (GVs), plug-in electric vehicles (PEVs) are vehicles relying primarily or exclusively on electricity and designed for being recharged by plugging its battery into the electric grid. Those plug-in vehicles equipped with both gasoline engines and electric motors (e.g., BYD F3DM, Chevrolet Volt, and Toyota Prius,[1] to name a few) are plug-in hybrid electric vehicles (PHEVs), while those relying entirely on electricity (e.g., BYD E6, Ford Focus EV, Mitsubishi i-MiEV, Nissan Leaf, Smart ED, and Subaru R1E) are classified as battery electric vehicles (BEVs) or all electric vehicles (AEVs).

Once a BEV's battery storage is depleted, the vehicle cannot be driven any further. Among the aforementioned BEV models, the driving range of a Mitsubishi i-MiEV of its 2011 version is around 62 miles, while a Nissan Leaf can run up to

[1] Note that Toyota Prius includes a family of vehicle models with different engine types and technologies, one of which is a plug-in hybrid electric version.

100 miles on a single charge, for example. Given that the electricity consumption is typically proportional to the driving distance, the driving range for BEVs is primarily confined by the battery capacity. At the initial stage of the market, the number of charging stations in most urban areas at present is very limited (for instance, among all the states in US, California is now the only state that has more than five hundreds charging stations (DOE 2011)), which makes drivers perceive the driving range limit as a potential worry. This phenomenon has been described as the so-called "range anxiety": the mental distress or fear of being stranded because the battery runs out of charge (Mock et al. 2010). Although many cities are planning the construction and expansion of charging infrastructures for BEVs, it is susceptibly likely that in the foreseeable future BEV commuters will need to charge their vehicles at home most of the time (Marrow et al. 2008). It is obvious that the driving range limit inevitably adds a certain level of restrictions to BEV drivers' travel behaviors, at least in a long future period prior to the coverage of recharging infrastructures reaching a sufficient level.

The widespread adoption of PEVs calls for fundamental changes to the existing network flow modeling tools for properly capturing changed behaviors and induced constraints in forecasting travel demands and evaluating transportation development plans. This paper is devoted to developing a network equilibrium model that takes into account the driving range limit of BEVs and implementing the model for evaluating the impacts of the driving range limit on destination, route and parking choice behaviors under equilibrium conditions. As we will see, given the simple settings for the driving range limit and multi-dimensional travel choice set, this network equilibrium problem can be characterized by a convex optimization model, and be easily incorporated into the existing trip-based travel demand forecasting framework that has been extensively used across transportation planning agencies.

The remaining part of this paper is organized as follows. The relevant literature is reviewed in the section of "Literature review". The model formulation and solution properties are elaborated in section "Model formulation and analysis". In section "Solution approach", the partial linearization method is adapted for solving this problem. Section "Numerical analysis" presents and analyzes the computational results for a couple of numerical examples of small and medium sizes. Finally, we conclude the paper and sketch our future research directions.

Literature review

The network equilibrium problem we will address below synthesizes a few protrudent modeling features compared to other problems in the literature. Specifically, our problem implies a simultaneous determination of destination, route and parking choices of both GV and BEV drivers, considers different parking policies and infrastructure availability for GVs and BEVs, and imposes a limit on the driving distance of BEVs in a combined urban traffic routing-parking equilibrium system. Given the inclusion of these modeling elements, our review work below is accordingly streamlined by tracing a number of relevant problem classes in the wide context of network equilibrium.

Network equilibrium problems with combined travel choices

Network equilibrium models with combined travel choices have been extensively studied in past decades, and there is a substantial body of literature on relevant problems of destination and route choices.

In the travel forecasting context, combined network equilibrium or travel demand models were developed for overcoming the inherent inconsistency of the sequential procedure. If travel cost functions are separable across different links, these models can be in general formulated as convex optimization models. Florian et al. (1975) and Evans (1976) separately proposed combined trip distribution and traffic assignment models, as an extension of the convex optimization formulation for the prime traffic assignment problem by Beckmann et al. (1956). In their work, the traffic assignment process follows the user-equilibrium principle and the trip distribution module follows a gravity model. A partial linearization algorithm was proposed by Evans (1976) to solve the combined model, which linearizes only the route choice part of the objective function. To capture the congestion effect at destinations, Oppenheim (1993) added the endogenous destination cost into the above model, as a function of the arriving flow at destinations. Erlander (1990) derived an alternative combined trip distribution and traffic assignment problem, the unique feature of which is that the stochastic user equilibrium instead of deterministic user equilibrium is used for specify the traffic assignment pattern. The same model appears in Lundgren and Patriksson (1998), who made an algorithmic progress by combining the partial linearization algorithm with the disaggregate simplicial decomposition algorithm for updating link flows, which significantly improves the computational performance of the original partial linearization method by Evans.

When multiple user classes (i.e., travelers with different behavioral or choice characteristics) are considered, a prerequisite for the integrability condition on the objective function of an optimization model is that the cross-class flow-cost effects must be symmetric (Dafermos 1972). Florian and Nguyen (1978) developed a combined trip distribution, modal split and trip assignment model which removes the asymmetric Jacobian elements by using separate traffic and transit subnetworks. Friesz (1981) presented an equivalent optimization model for combined multiclass trip distribution, traffic assignment and modal split, which avoided the symmetry restriction on cost functions by expressing Wardrop's user-equilibrium principle as a set of nonlinear constraints. However, the resulting optimization model is not convex and requires route enumeration for its solution. Lam and Huang (1992) proposed a convex formulation for the multiclass combined trip distribution and traffic assignment problem which uses symmetric "normalized" link cost functions. They adapted the Frank-Wolfe algorithm and Evans algorithm for solving the problem and concluded that the Evans algorithm performs better than the Frank-Wolfe. Wong et al. (2004) presented a combined trip distribution, (hierarchical) modal split, and traffic assignment model with multiple user and mode classes and employed the Evans algorithm for its solutions. In a different modeling paradigm, Ho et al. (2006) proposed a combined trip distribution and traffic assignment model for a continuum traffic equilibrium problem with multiple user classes. They assumed that route choice follows the user-equilibrium principle and destination choice is specified by a logit model. A finite-element method was used to solve the model.

Network equilibrium problems with parking choices

Conventionally, parking choice is not included in combined urban network equilibrium or travel demand models. However, an increasing number of studies have found that parking becomes a major contributing factor to urban traffic congestion (e.g., Axhausen and Polak 1991; Lambe 1996; Anderson and de Palma 2004). Very often, drivers spend a significant portion of their total travel time looking for available parking spaces and circling for parking considerably worsens traffic conditions and air quality of urban networks that are already congested. For example, circling on streets for parking contributes to up to 30 % of San Francisco's congestion (SFMTA 2011). Due to this reason, in many urban areas, parking management has become an essential part of urban transportation management and should be used as a tool to mitigate traffic congestion and balance land development. Preferential parking policies, for example, may be implemented to encourage the usage of low-emission vehicles. As an integral part of urban network modeling and analysis, it is important to understand and predict the parking choice behaviors of travelers under various parking policies and travel choice restrictions.

Many studies on static or dynamic network equilibrium analysis of combined parking and travel choice models have been carried out in the literature. Here we highlight a few examples. Florian and Los (1980) proposed an entropy maximization-based model for predicting parking occupation in a park-and-ride network. Parking capacity was explicitly considered as a side constraint in their model. Nour Eldin et al. (1981) developed a combined parking and traffic assignment model which takes into account the interaction between parking supply and traffic flows by adding imaginary links to represent parking-related searching, parking and walking activities. Gur and Beimborn (1984) modeled the time spending in looking or waiting for a parking space as an increasing function of the utilization level of the parking area, and analyzed the parking process in the framework of user equilibrium. In view of that static models are unable to characterize the spatiotemporal travel and parking interactions, Bifulco (1993) developed a quasi-dynamic network equilibrium model in which the traffic flow pattern in the road network during each time interval is in a steady-state equilibrium with stochastic perceptions while the parking demand in the parking system has an accumulated effect across time intervals. Specifically, it is assumed in his research that travelers start and complete their journeys within the same time interval and the connection between successive time intervals is represented by the parking occupancy that is carried over to the next interval. In a subsequent study, Lam et al. (2006) also presented a more complex quasi-dynamic network equilibrium model which simultaneously considers departure time, route, parking location and parking duration choices in road networks with multiple user classes and multiple parking facilities. Their model allows trips to go through multiple time intervals.

Network equilibrium problems with side constraints

From the modeling perspective, the driving range limit of BEVs poses a side constraint on their path lengths. By side constraints, we mean those extra constraints imposed on traffic flows or travel costs of links, paths, origin–destination (O-D) pairs, origins, or destinations, in addition to the basic flow conservation and nonnegativity constraints. Adding side constraints into a network equilibrium model often increases the solution difficulty significantly. In the literature, research efforts on network equilibrium problems with side constraints were dominantly focused on side constraints for link flows, which lead to the so-called capacity-constrained traffic assignment problem (e.g., Hearn 1980; Larsson and Patriksson 1995; Nie et al. 2004). This type of problems has been well addressed by dual, penalty or Lagrangian relaxation methods. Larsson and Patriksson (1999) extended the capacity-constrained traffic assignment problem to a more general form, but still assumed that the general side constraints are imposed on links. In the class of dynamic traffic assignment models, the link capacity is in general explicitly included to confine the traffic throughout rate and the queue formation and dissipation.

In contrast, few research activities have been devoted to traffic assignment problems with path-level side constraints, except two recent studies. Jahn et al. (2005) proposed a system-optimal traffic assignment problem with an upper bound on path travel times for designing a route guidance system that simultaneously promotes system optimum and user fairness; Jiang et al. (2012) presented a user-equilibrium problem with an upper bound on path lengths for the need of modeling the route choice pattern of electric vehicles. Both of the research teams solved their problems in the Frank-Wolfe solution framework. Despite the different settings of the optimization objective (i.e., system optimum vs. user equilibrium) and path-constrained component (i.e., flow-dependent travel time vs. flow-independent travel distance), the two problems pose a similar path cost structure under optimality conditions. In particular, Jiang et al. (2012) showed that the optimal path cost of an individual traveler with a path length limit contains a special path-specific out-of-limit cost term.

Model formulation and analysis

It is most likely that in a long period in the future GVs and BEVs will coexist in the automobile market. For this reason, the model we present below includes two classes of vehicles, GVs and BEVs, which distinguish from each other in terms of driving distance range and travel cost composition. To simplify the modeling complexity in such a mixed-traffic network and ensure our modeling focus on the most essential demand characteristics and system behaviors of the anticipated network equilibrium, a set of assumptions regarding demand heterogeneity, travel behavior, and parking availability are suggested here.

First, it is assumed that the demand population is only comprised of GVs and BEVs. PHEVs are not explicitly considered since they can be simply treated as an in-between class of GVs and BEVs in terms of the technological and economic features (i.e., driving range limit and travel cost composition), or a special type of GVs with lower operating costs. For modeling simplicity, we consider only one type of BEVs. Certainly, if needed, multiple types of BEVs with different driving range limits and unit operating costs can be easily incorporated into the model without changing the problem's nature and model's structure.

Second, we assume that the total travel demand at each origin for each class/type of vehicles is deterministically known a priori. In other words, we ignore the elastic and stochastic effects of demand generation. For route choice, the most prime form of traffic assignment—user equilibrium—is used, in which each traveler chooses a route that minimizes his/her travel cost and no one can reduce his/her travel cost by unilaterally switching to an alternative route. For an individual GV traveler, user equilibrium simply implies a minimum cost search; for a BEV traveler, however, it poses a distance-constrained minimum cost problem. On the other hand, individual destination and parking choices are specified by the multinomial logit model (Ben-Akiva and Lerman 1985), which is in principle equivalent to the traditionally used gravity model for trip distribution (Anas 1983). In the network equilibrium context, this specification also results in the stochastic user-equilibrium state on the network level when the cost-flow consistency is required. Again, subject to the added driving range limit, the aggregated destination and parking flow patterns will be shifted for BEV travelers.

Third, we assume that two types of parking garages are available at each destination, namely, ordinary parking garages and special parking garages. Special parking garages are equipped with charging stations and are only open to BEVs, while ordinary parking garages are open to both GVs and BEVs. For such a parking classification, on one hand, it reflects the emerging parking policies that encourage the purchase and usage of BEVs; on the other hand, it offers a mechanism of preserving the charging equipment and devices exclusively for BEVs, if needed. Despite the fact that there remain multiple ordinary and special parking garages at each destination, and both ordinary and special parking spots exist in a single garage, we aggregate all ordinary parking spaces at each destination into one virtual ordinary parking facility and all special parking spaces into one virtual special parking facility. Explicitly treating multiple ordinary and special parking garages only slightly adds the modeling complexity while not changing the problem's structure.

Finally, without loss of generality, we assume that both GV and BEV travelers use a common form of systematic travel cost/disutility for determining their travel and parking choices: travel time (i.e., driving time and parking access and search time) + monetary cost (i.e., operating cost and parking charge). While the travel time and parking time are assumed to be the same to both the GV and BEV classes (along the same route and in the same parking facility), their operating costs considerably differ due to the difference between the gasoline price and electricity price and possible different road and parking pricing policies for GVs and BEVs. As we will see, this cost difference, plus the driving range limit and parking infrastructure availability, will significantly distinguish GV and BEV travelers in their decision making behaviors in face of destination, route and parking choices.

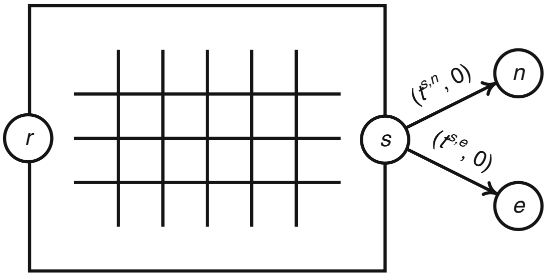
It is clear that given the set of modeling assumptions described above, the complexity of the proposed model is primarily due to the introduction of BEVs into the combined network equilibrium problem. In particular, the destination and route choices of BEV travelers are subject to the driving range limit, and they face parking choices between the special and ordinary parking facilities. If the travel and parking flow patterns of BEVs are fixed, the remaining modeling issues for GVs can be simply aggregated as a combined trip distribution and traffic assignment problem, which, as we discussed earlier, has been well addressed by Evans (1976), Florian et al. (1975), Erlander (1990), Oppenheim (1993), and Lundgren and Patriksson (1998), among others. The purpose of this paper is on the development of a distance-constrained, combined network equilibrium model for estimating the aggregated behavior of an equilibrium transportation network with both GVs and BEVs and on an investigation of the impacts of the battery capacity and penetration rate of BEVs on the network performance. It is expected that the battery capacity and other vehicle performance specifications will be continuously improved in future years to accelerate the growth of the penetration rate of BEVs in urban networks.

The remaining part of this section will discuss the model formulation and solution properties. For discussion convenience, we refer readers to the notation list presented at the beginning of this paper.

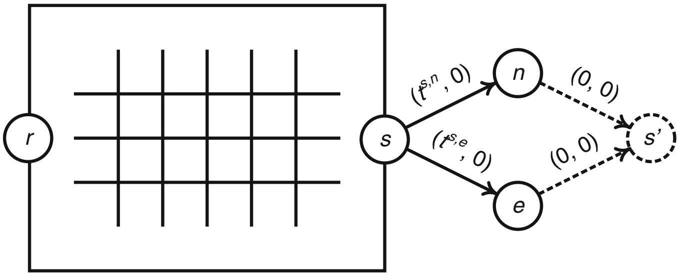
Model formulation

The general network representation paradigm, where nodes denote origins, destinations and intersections, and links denote roadway segments between nodes, is extended to include parking facilities as follows: Every destination node s is connected with two parking nodes via two parking links that represent the ordinary parking facility and special parking facility at this destination, respectively. The parking access and search time, the parking fee, and other attributes of the parking facilities are all associated with the parking links, as shown in Fig. 1a. Under this representation, all infrastructure elements in this integrated roadway and parking system are consistently depicted in a node-link network, in which the congestion effect of each parking facility can be modeled the same way as that of a roadway link. Moreover, we add a dummy destination node for any O-D pair in the network and a dummy link with zero travel cost and travel distance between each parking node to the dummy node, as shown in Fig. 1b. Though it is not necessary from the modeling perspective, this node and link additions let us treat parking choice and route choice in a unified modeling mechanism and creates great algorithmic convenience, as we will see later on.

Given the above network representation, the combined trip distribution, traffic assignment and parking split problem can be written into the following convex optimization model:



(a) An O-D pair



(b) An O-D pair with a dummy destination node

Fig. 1 Network representation

$$\begin{aligned}\min z(\mathbf{x}(\mathbf{f}),\mathbf{q}) = {} & \sum_{rs}\left\{\frac{1}{\gamma_{\mathrm{g}}}q_{\mathrm{g}}^{rs}\left[\ln\left(q_{\mathrm{g}}^{rs}\right)-1\right]+\frac{1}{\gamma_{\mathrm{e}}}q_{\mathrm{e}}^{rs}\left[\ln\left(q_{\mathrm{e}}^{rs}\right)-1\right]\right\} \\ & +\rho\sum_{a}\int_0^{x_a}t_a(\omega)d\omega+\sum_a\left(x_{a,\mathrm{g}}u_{\mathrm{g}}d_a+x_{a,\mathrm{e}}u_{\mathrm{e}}d_a\right) \\ & +\rho\sum_s\left(\int_0^{R^{s,n}}t^{s,n}(v)dv+\int_0^{R^{s,e}}t^{s,e}(v)dv\right) \\ & +\sum_s\left(R^{s,n}c^{s,n}+R^{s,e}c^{s,e}\right)\end{aligned} \tag{1}$$

subject to

$$\sum_k f_{k,\mathrm{g}}^{rs,n}=q_{\mathrm{g}}^{rs}\quad\left(\pi_{\mathrm{g}}^{rs}\right)\quad\forall r,s \tag{2}$$

$$\sum_k\left(f_{k,\mathrm{e}}^{rs,n}+f_{k,\mathrm{e}}^{rs,e}\right)=q_{\mathrm{e}}^{rs}\quad\left(\pi_{\mathrm{e}}^{rs}\right)\quad\forall r,s \tag{3}$$

$$\sum_s q_{\mathrm{g}}^{rs}=O_{\mathrm{g}}^{r}\quad\left(\mu_{\mathrm{g}}^{r}\right)\quad\forall r \tag{4}$$

$$\sum_s q_{\mathrm{e}}^{rs}=O_{\mathrm{e}}^{r}\quad\left(\mu_{\mathrm{e}}^{r}\right)\quad\forall r \tag{5}$$

$$\left(D-\sum_{a} d_a \delta_{a,k}^{rs}\right)\left(f_{k,\mathrm{e}}^{rs,n}+f_{k,\mathrm{e}}^{rs,e}\right) \geq 0 \quad \left(\lambda_k^{rs}\right) \quad \forall r,s,k \tag{6}$$

$$f_{k,\mathrm{g}}^{rs,n}, f_{k,\mathrm{e}}^{rs,n}, f_{k,\mathrm{e}}^{rs,e} \geq 0 \quad \forall r,s,k \tag{7}$$

where

$$x_a = x_{a,\mathrm{g}} + x_{a,\mathrm{e}} \quad \forall a \tag{8}$$

$$x_{a,\mathrm{g}} = \sum_{rs}\sum_{k} f_{k,\mathrm{g}}^{rs,n}\delta_{a,k}^{rs} \quad \forall a \tag{9}$$

$$x_{a,\mathrm{e}} = \sum_{rs}\sum_{k}\left(f_{k,\mathrm{e}}^{rs,n}\delta_{a,k}^{rs}+f_{k,\mathrm{e}}^{rs,e}\delta_{a,k}^{rs}\right) \quad \forall a \tag{10}$$

$$R^{s,n} = \sum_{r}\sum_{k}\left(f_{k,\mathrm{g}}^{rs,n}+f_{k,\mathrm{e}}^{rs,n}\right) \quad \forall s \tag{11}$$

$$R^{s,e} = \sum_{r}\sum_{k} f_{k,\mathrm{e}}^{rs,e} \quad \forall s \tag{12}$$

where the variables in the parentheses beside equations (2)–(6) are dual variables associated with constraints.

The objective function of the optimization model can be briefly described as follows. Its first term specifies the destination choice behavior, which will be proved to follow the multinomial logit model. The second term replicates Beckmann's transformation for the user-equilibrium traffic assignment. The third term is the sum of the operation costs, which are proportional to travel distances and independent of traffic flow rates. The combination of the second and third components ensures that the route choice in our model follows the Wardropian user-equilibrium principle in terms of the total individual travel cost (i.e., sum of individual travel time and individual monetary cost). The fourth term is used to indicate the constrained user-equilibrium parking demand split, which has a similar form to the second term, where the constraint is from the parking restriction that all special parking facilities are not available for GVs. Similar to the third term, the fifth term simply denotes the sum of flow-independent parking costs.

The constraint set of the model can be grouped into three subsets. Constraints (2)–(5) are flow conservation equations. Constraint (6) defines the distance constraint for feasible paths of BEVs, i.e., if the BEV flow rate on path k connecting origin r and destination s is positive, the length of this path cannot exceed the driving range limit D. The set of constraints (7) simply define the non-negativity of all flow variables. In addition, definitional constraints (8)–(12) specify the relationship among link flows, path flows, and parking demands.

The link travel time functions t_a are assumed to be separable between different network links and identical for different vehicle classes. These functions are assumed to be positive, monotonically increasing, and strictly convex. The same assumptions are also held for the parking access and search time functions $t^{s,n}$ and $t^{s,e}$. These assumptions on the link travel time and parking time functions can be mathematically written as follows. Specifically, for any network link a, if $x_a \geq 0$, we have

$$t_a(x_a) > 0, \quad \frac{dt_a(x_a)}{dx_a} > 0, \quad \frac{\partial t_a(x_a)}{\partial x_b} = 0 \quad \text{for}\, b \neq a, \text{ and } \frac{d^2 t_a(x_a)}{dx_a^2} > 0$$

and for any parking lot s, if $R^{s,n} \geq 0$ and $R^{s,e} \geq 0$, we have

$$t^{s,n}(R^{s,n}) > 0, \quad \frac{dt^{s,n}(R^{s,n})}{dR^{s,n}} > 0, \quad \frac{\partial t^{s,n}(\cdot)}{\partial R^{m,n}} = 0 \quad \text{for}\, s \neq m, \quad \text{and } \frac{d^2 t^{s,n}(R^{s,n})}{d(R^{s,n})^2} > 0$$

$$t^{s,e}(R^{s,e}) > 0, \quad \frac{dt^{s,e}(R^{s,e})}{dR^{s,e}} > 0, \quad \frac{\partial t^{s,e}(\cdot)}{\partial R^{m,e}} = 0 \quad \text{for}\, s \neq m, \quad \text{and } \frac{d^2 t^{s,e}(R^{s,e})}{d(R^{s,e})^2} > 0$$

The following part then discusses the solution existence and uniqueness of the solution of the proposed model and its equivalence to the proposed network equilibrium conditions.

Solution existence and uniqueness

The presence of the distance constraint (6) might result in an infeasible problem, when none of the paths from an origin to all its destinations satisfies the distance constraint. When the problem is feasible, given that the feasible region confined by constraints (2)–(12) is compact, we know that optimal solutions must exist. Further, given the strict convexity of the link travel time functions, the parking access and search time functions, and the entropy term in the objective function for destination choice are all strictly convex, ensuring that the objective function in the feasible region, it can be readily proved that the Hessian of the objective function (1) is positive definite. Moreover, the feasible region defined by linear constraints (2)–(12) is convex. Therefore, the optimization problem defined in (1)–(12) poses a unique solution for destination flows and link flows (but not necessarily for GV and BEV link flows).

Solution equivalence

The equilibrium conditions corresponding to the optimal solution of the proposed convex optimization problem can be analyzed below through checking the first-order conditions of its Lagrangian dual problem. First, we write the Lagrangian dual problem as follows, given the dual variables of constraints, which are listed in the parentheses beside equations (2)–(6),

$$\begin{aligned} L = z(\mathbf{x}(\mathbf{f}), \mathbf{q}) &+ \sum_{rs} \pi_{\mathrm{g}}^{rs} \left[q_{\mathrm{g}}^{rs} - \sum_k f_{k,\mathrm{g}}^{rs,n} \right] + \sum_{rs} \pi_{\mathrm{e}}^{rs} \left[q_{\mathrm{e}}^{rs} - \sum_k \left(f_{k,\mathrm{e}}^{rs,n} + f_{k,\mathrm{e}}^{rs,e} \right) \right] \\ &+ \sum_r \mu_{\mathrm{g}}^r \left(O_{\mathrm{g}}^r - \sum_s q_{\mathrm{g}}^{rs} \right) + \sum_r \mu_{\mathrm{e}}^r \left(O_{\mathrm{e}}^r - \sum_s q_{\mathrm{e}}^{rs} \right) \\ &- \sum_{rs} \sum_k \lambda_k^{rs} \left(D - \sum_a d_a \delta_{a,k}^{rs} \right) \left(f_{k,\mathrm{e}}^{rs,n} + f_{k,\mathrm{e}}^{rs,e} \right) \end{aligned} \quad (13)$$

The solution equivalence between the original problem and the Lagrangian dual problem can be realized if the dual problem is maximized with respect to the dual variables, in which the relaxed Lagrangian problem is minimized subject to the remaining flow nonnegativity constraints:

$$f_{k,\mathrm{g}}^{rs,n}, \quad f_{k,\mathrm{e}}^{rs,n}, \quad f_{k,\mathrm{e}}^{rs,e} \geq 0 \quad \forall r, s, k \tag{14}$$

Among the dual variables, the one associated with the distance constraint is the only constrained dual variable:

$$\lambda_k^{rs} \geq 0 \quad \forall r, s, k \tag{15}$$

The derivatives of the Lagrangian dual problem with respect to path flow variables are,

$$\frac{\partial L}{\partial f_{k,\mathrm{g}}^{rs,n}} = \rho \sum_a t_a(x_a)\delta_{a,k}^{rs} + \sum_a u_\mathrm{g} d_a \delta_{a,k}^{rs} + \rho t^{s,n}(R^{s,n}) + c^{s,n} - \pi_\mathrm{g}^{rs} \tag{16}$$

$$\frac{\partial L}{\partial f_{k,\mathrm{e}}^{rs,n}} = \rho \sum_a t_a(x_a)\delta_{a,k}^{rs} + \sum_a u_\mathrm{e} d_a \delta_{a,k}^{rs} + \rho t^{s,n}(R^{s,n}) + c^{s,n} - \pi_\mathrm{e}^{rs} - \lambda_k^{rs}\left(D - \sum_a d_a \delta_{a,k}^{rs}\right) \tag{17}$$

$$\frac{\partial L}{\partial f_{k,\mathrm{e}}^{rs,e}} = \rho \sum_a t_a(x_a)\delta_{a,k}^{rs} + \sum_a u_\mathrm{e} d_a \delta_{a,k}^{rs} + \rho t^{s,e}(R^{s,e}) + c^{s,e} - \pi_\mathrm{e}^{rs} - \lambda_k^{rs}\left(D - \sum_a d_a \delta_{a,k}^{rs}\right) \tag{18}$$

the derivative with respect to dual variable λ_k^{rs} is,

$$\frac{\partial L}{\partial \lambda_k^{rs}} = -\left(D - \sum_a d_a \delta_{ak}^{rs}\right)\left(f_{k,\mathrm{e}}^{rs,n} + f_{k,\mathrm{e}}^{rs,e}\right) \tag{19}$$

and the derivatives with respect to O-D flow variables are,

$$\frac{\partial L}{\partial q_\mathrm{g}^{rs}} = \frac{1}{\gamma_\mathrm{g}} \ln\left(q_\mathrm{g}^{rs}\right) + \pi_\mathrm{g}^{rs} - \mu_\mathrm{g}^{r} \tag{20}$$

$$\frac{\partial L}{\partial q_\mathrm{e}^{rs}} = \frac{1}{\gamma_\mathrm{e}} \ln\left(q_\mathrm{e}^{rs}\right) + \pi_\mathrm{e}^{rs} - \mu_\mathrm{e}^{r} \tag{21}$$

Now, let us set,

$$\rho t_a(x_a) + u_\mathrm{g} d_a = p_{a,\mathrm{g}} \tag{22}$$

$$\rho t_a(x_a) + u_\mathrm{e} d_a = p_{a,\mathrm{e}} \tag{23}$$

where $p_{a,\mathrm{g}}$ and $p_{a,\mathrm{e}}$ are the generalized link travel costs for GVs and BEVs, respectively, each of which includes two cost components: travel time and operation cost. Moreover, we further set,

$$\rho \sum_a t_a(x_a)\delta_{a,k}^{rs} + \sum_a u_\mathrm{g} d_a \delta_{a,k}^{rs} = \sum_a p_{a,\mathrm{g}} \delta_{a,k}^{rs} = p_{k,\mathrm{g}}^{rs} \tag{24}$$

$$\rho \sum_a t_a(x_a)\delta^{rs}_{a,k} + \sum_a u_e d_a \delta^{rs}_{a,k} = \sum_a p_{a,e}\delta^{rs}_{a,k} = p^{rs}_{k,\mathrm{e}} \tag{25}$$

where $p^{rs}_{k,\mathrm{g}}$ and $p^{rs}_{k,\mathrm{e}}$ are the generalized path travel cost for GVs and BEVs, respectively. Let $\sum_a d_a \delta^{rs}_{ak} = l^{rs}_k$, then l^{rs}_k is the distance of path k from r to s. Then, the first-order conditions of the Lagrangian dual problem are thus,

$$f^{rs,n*}_{k,\mathrm{g}}\left[p^{rs*}_{k,\mathrm{g}} + \rho t^{s,n*} + c^{s,n} - \pi^{rs*}_{\mathrm{g}}\right] = 0 \quad \forall r,s,k \tag{26}$$

$$p^{rs*}_{k,\mathrm{g}} + \rho t^{s,n*} + c^{s,n} - \pi^{rs*}_{\mathrm{g}} \geq 0 \quad \forall r,s,k \tag{27}$$

$$f^{rs,n*}_{k,\mathrm{e}}\left[p^{rs*}_{k,\mathrm{e}} + \rho t^{s,n*} + c^{s,n} - \lambda^{rs*}_k\left(D - l^{rs}_k\right) - \pi^{rs*}_{\mathrm{e}}\right] = 0 \quad \forall r,s,k \tag{28}$$

$$p^{rs*}_{k,\mathrm{e}} + \rho t^{s,n*} + c^{s,n} - \lambda^{rs*}_k\left(D - l^{rs}_k\right) - \pi^{rs*}_{\mathrm{e}} \geq 0 \quad \forall r,s,k \tag{29}$$

$$f^{rs,n*}_{k,\mathrm{e}}\left[p^{rs*}_{k,\mathrm{e}} + \rho t^{s,e*} + c^{s,e} - \lambda^{rs*}_k\left(D - l^{rs}_k\right) - \pi^{rs*}_{\mathrm{e}}\right] = 0 \quad \forall r,s,k \tag{30}$$

$$p^{rs*}_{k,\mathrm{e}} + \rho t^{s,e*} + c^{s,e} - \lambda^{rs*}_k\left(D - l^{rs}_k\right) - \pi^{rs*}_{\mathrm{e}} \geq 0 \quad \forall r,s,k \tag{31}$$

$$\frac{1}{\gamma_{\mathrm{g}}}\ln\left(q^{rs*}_{\mathrm{g}}\right) + \pi^{rs*}_{\mathrm{g}} - \mu^{r*}_{\mathrm{g}} = 0 \quad \forall r,s \tag{32}$$

$$\frac{1}{\gamma_{\mathrm{e}}}\ln\left(q^{rs*}_{\mathrm{e}}\right) + \pi^{rs*}_{\mathrm{e}} - \mu^{r*}_{\mathrm{e}} = 0 \quad \forall r,s \tag{33}$$

$$\lambda^{rs*}_k\left(D - l^{rs}_k\right)\left(f^{rs,n*}_{k,\mathrm{e}} + f^{rs,e*}_{k,\mathrm{e}}\right) = 0 \quad \forall r,s,k \tag{34}$$

$$\left(D - l^{rs}_k\right)\left(f^{rs,n*}_{k,\mathrm{e}} + f^{rs,e*}_{k,\mathrm{e}}\right) \geq 0 \quad \forall r,s,k \tag{35}$$

$$q^{rs*}_{\mathrm{g}} - \sum_k f^{rs,n*}_{k,\mathrm{g}} = 0 \quad \forall r,s \tag{36}$$

$$q^{rs*}_{\mathrm{e}} - \sum_k\left(f^{rs,n*}_{k,\mathrm{e}} + f^{rs,e*}_{k,\mathrm{e}}\right) = 0 \quad \forall r,s \tag{37}$$

$$O^r_{\mathrm{g}} - \sum_s q^{rs*}_{\mathrm{g}} = 0 \quad \forall r \tag{38}$$

$$O^r_{\mathrm{e}} - \sum_s q^{rs*}_{\mathrm{e}} = 0 \quad \forall r \tag{39}$$

$$f^{rs,n*}_{k,\mathrm{g}}, \quad f^{rs,n*}_{k,\mathrm{e}}, \quad f^{rs,e*}_{k,\mathrm{e}} \geq 0 \quad \forall r,s,k \tag{40}$$

$$\lambda^{rs*}_k \geq 0 \quad \forall r,s,k \tag{41}$$

Route and parking choices

For combined route and parking choices, optimality conditions (26)–(31) plus (40) together can be rewritten as the following three systems of conditional equations/ inequalities:

$$\begin{cases} p_{k,\mathrm{g}}^{rs*} + \rho t^{s,n*} + c^{s,n} = \pi_{\mathrm{g}}^{rs*} & \text{if} \quad f_{k,\mathrm{g}}^{rs,n*} > 0 \\ p_{k,\mathrm{g}}^{rs*} + \rho t^{s,n*} + c^{s,n} \geq \pi_{\mathrm{g}}^{rs*} & \text{if} \quad f_{k,\mathrm{g}}^{rs,n*} = 0 \end{cases} \tag{42}$$

$$\begin{cases} p_{k,\mathrm{e}}^{rs*} + \rho t^{s,n*} + c^{s,n} + \lambda_k^{rs*}\left(l_k^{rs} - D\right) = \pi_{\mathrm{e}}^{rs*} & \text{if} \quad f_{k,\mathrm{e}}^{rs,n*} > 0 \\ p_{k,\mathrm{e}}^{rs*} + \rho t^{s,n*} + c^{s,n} + \lambda_k^{rs*}\left(l_k^{rs} - D\right) \geq \pi_{\mathrm{e}}^{rs*} & \text{if} \quad f_{k,\mathrm{e}}^{rs,n*} = 0 \end{cases}$$
$$\begin{cases} p_{k,\mathrm{e}}^{rs*} + \rho t^{s,n*} + c^{s,e} + \lambda_k^{rs*}\left(l_k^{rs} - D\right) = \pi_{\mathrm{e}}^{rs*} & \text{if} \quad f_{k,\mathrm{e}}^{rs,e*} > 0 \\ p_{k,\mathrm{e}}^{rs*} + \rho t^{s,n*} + c^{s,e} + \lambda_k^{rs*}\left(l_k^{rs} - D\right) \geq \pi_{\mathrm{e}}^{rs*} & \text{if} \quad f_{k,\mathrm{e}}^{rs,e*} = 0 \end{cases} \tag{43}$$

In the first system of equations/inequalities (42), π_{g}^{rs*} can be interpreted as the minimum combined travel and parking cost for GVs from origin r to destination s. We name it the *minimum composite path cost for GVs*. Similarly, π_{e}^{rs*}, the *minimum composite path cost for BEVs*, can be interpreted as the minimum travel and parking cost for BEVs from origin r to destination s.

In the second and third systems, note that different from that for GVs, the composite path cost for BEVs includes the term $\lambda_k^{rs*}(l_k^{rs} - D)$, which is defined as the *path out-of-range cost* (Jiang et al. 2012). The entire set of equilibrium conditions given above specified that the combined route and parking choice behaviors of gasoline and electric vehicles follow the user-equilibrium principle, which can be respectively interpreted as follows,

- If the generalized path travel cost of driving GVs on a path connecting r and s plus the parking cost at the regular parking facility at s is higher than the minimum composite path cost for GVs between r and s, no GV will choose this path.
- If the generalized path travel cost of driving BEVs on a path connecting r and s plus the parking cost at the regular parking facility at s and its out of range cost along this path is higher than the minimum composite path cost for BEVs between r and s, no BEV will choose this path and park at the regular parking facility.
- If the generalized path travel cost of driving BEVs on a path connecting r and s plus the parking cost at the special parking facility at s and its out-of-range cost along this path is higher than the minimum composite path cost for BEVs between r and s, no BEV will choose this path and park at the special parking facility.

We can also rewrite the equilibrium conditions in (34)–(35) and (41) in a similar way,

$$\begin{cases} \left(D - l_k^{rs}\right)\left(f_{k,\mathrm{e}}^{rs,n*} + f_{k,\mathrm{e}}^{rs,e*}\right) = 0 & \text{if} \quad \lambda_k^{rs*} > 0 \\ \left(D - l_k^{rs}\right)\left(f_{k,\mathrm{e}}^{rs,n*} + f_{k,\mathrm{e}}^{rs,e*}\right) \geq 0 & \text{if} \quad \lambda_k^{rs*} = 0 \end{cases} \tag{44}$$

which can be interpreted as the following three cases:

- If $l_k^{rs} > D$, the total flow of BEVs along path k, $f_{k,\mathrm{e}}^{rs,n*} + f_{k,\mathrm{e}}^{rs,e*}$, equals zero. That is, if the length of a path exceeds the driving range limit of BEVs, no BEV will choose this path.

- If $l_k^{rs} < D$ and $f_{k,\mathrm{e}}^{rs,n*} + f_{k,\mathrm{e}}^{rs,e*} > 0$, we must have $\lambda_k^{rs*} = 0$. As a result, the out-of-range-cost of path k, $\lambda_k^{rs*}(l_k^{rs} - D) = 0$. That is, no extra cost will be incurred if the length of a path is shorter than the driving range limit.
- If $l_k^{rs} = D$, we know that condition $\lambda_k^{rs*}(l_k^{rs} - D) = 0$ will always hold and no extra out-of-range cost will be incurred on this path.

Despite the route and parking equilibrium conditions are presented above in a combined form, the two choices can be explicitly represented in separate equilibrium regimes, due to the existence of a single destination node between any O-D pair and the parking links connected with this destination node (see Fig. 1a). If we add a dummy destination node for this O-D pair and a dummy link with zero travel cost between each parking node to the dummy node (see Fig. 1b), it is readily known that the subnetwork connecting the origin and destination nodes and the parking subnetwork connecting the destination node and the dummy destination node both hold an equilibrium state under the optimality conditions. With the added dummy node, the combined route and parking equilibrium collapses to a pure Wardropian user equilibrium for routing. Therefore, the route equilibrium and parking equilibrium can be separated, given that the Wardropian user equilibrium is one of the network equilibrium conditions that imply the Markovian routing behavior (e.g., Akamatsu 1996; Correa and Stier-Moses 2011) in which any individual would choose his/her remaining route to the sink node (i.e., the dummy destination in our case) without considering the route he/she has experienced between the source node (i.e., the origin in our case) and his/her current location.

In particular, the equilibrium route and parking choices for GVs and BEVs can be briefly stated as follows. For each O-D pair, the routing choice of GVs follows the Wardropian user equilibrium while the routing choice of BEVs follows a distance-constrained Wardropian user equilibrium; the parking choice of GVs follows a facility-restricted Wardropian user equilibrium while the parking choice of BEVs follows the Wardropian user equilibrium.

Destination choice

We prove below that the destination choice behaviors of both GV and BEV drivers obey the result of the multinomial logit model. The equilibrium conditions (32) and (33) can be rewritten as,

$$q_{\mathrm{g}}^{rs*} = \exp\left[-\gamma_{\mathrm{g}}\left(\pi_{\mathrm{g}}^{rs*} - \mu_{\mathrm{g}}^{r*}\right)\right] \quad \forall r, s \tag{45}$$

$$q_{\mathrm{e}}^{rs*} = \exp\left[-\gamma_{\mathrm{e}}\left(\pi_{\mathrm{e}}^{rs*} - \mu_{\mathrm{e}}^{r*}\right)\right] \quad \forall r, s \tag{46}$$

Equations (45) and (46) can be interpreted as the demand models for GV and BEV trips between O-D pair r and s.Given the origin demand conservation constraint, equations (45) and (46) can be respectively substituted into equations (4) and (5) to give,

$$O_{\mathrm{g}}^{r} = \sum_{s} q_{\mathrm{g}}^{rs*} = \sum_{s} \exp\left[-\gamma_{\mathrm{g}}\left(\pi_{\mathrm{g}}^{rs*} - \mu_{\mathrm{g}}^{r*}\right)\right] \quad \forall r \tag{47}$$

$$O_{\mathrm{e}}^{r} = \sum_{s} q_{\mathrm{e}}^{rs*} = \sum_{s} \exp\left[-\gamma_{\mathrm{e}}\left(\pi_{\mathrm{e}}^{rs*} - \mu_{\mathrm{e}}^{r*}\right)\right] \quad \forall r \tag{48}$$

Consequently, the destination choice probability for a GV driver departing from origin r and choosing destination s is,

$$P_{\mathrm{g}}^{rs*} = \frac{q_{\mathrm{g}}^{rs*}}{O_{\mathrm{g}}^{r}} = \frac{\exp\left[-\gamma_{\mathrm{g}}\pi_{\mathrm{g}}^{rs*}\right]}{\sum_{s}\exp\left[-\gamma_{\mathrm{g}}\pi_{\mathrm{g}}^{rs*}\right]} \quad \forall r, s \tag{49}$$

Similarly, the destination choice probability for a BEV driver departing from origin r and choosing destination s,

$$P_{\mathrm{e}}^{rs*} = \frac{q_{\mathrm{e}}^{rs*}}{O_{\mathrm{e}}^{r}} = \frac{\exp\left[-\gamma_{\mathrm{e}}\pi_{\mathrm{e}}^{rs*}\right]}{\sum_{s}\exp\left[-\gamma_{\mathrm{e}}\pi_{\mathrm{e}}^{rs*}\right]} \quad \forall r, s \tag{50}$$

It is evident that both GV and BEV drivers exactly follow the behaviors specified by the multinomial logit model in their destination choices.

It should be noted that in the proposed combined network equilibrium model the driving range limit affects the trip distribution result as well. If, between some O-D pair r-s, there is no path whose length is less than or equal to the driving range limit, i.e., $l_k^{rs} > D$, $\forall k \in K_{rs}$, we know that $\lambda_k^{rs*} = +\infty, \forall k \in K_{rs}$ and hence

$$\pi_{\mathrm{e}}^{rs*} = \min_{k \in K_{rs}} \left\{ p_{k,\mathrm{e}}^{rs*} + c^{s,n*} - \lambda_k^{rs*}\left(D - l_k^{rs}\right), p_{k,\mathrm{e}}^{rs*} + c^{s,e*} - \lambda_k^{rs*}\left(D - l_k^{rs}\right) \right\} = +\infty.$$

In this case, following Equation (50), $P_{\mathrm{e}}^{rs*} = 0$. This result, as an extreme example, illustrates the impact of the driving range limit on destination choice.

Sensitivity analysis

One of the primary concerns of this paper is on the equilibrium analysis of the impacts of the driving range limit of BEVs on travel and parking choices. In other words, beyond the model development, our concern also includes the evaluation of the changes of destination, route and parking choice patterns due to the change of the driving range limit, which poses a sensitivity analysis problem.

For many network equilibrium problems, such a sensitivity analysis task can be done by both analytical and numerical methods. Tobin and Friesz (1988) first developed a variational inequality-based sensitivity analysis method for evaluating the impact of the perturbation of network supply and demand parameters on deterministic equilibrium network flows; following this seminal work, Yang (1997) and Leurent (1998) extended the sensitivity analysis approach to evaluating the changes of elastic-demand equilibrium networks and bi-criterion equilibrium networks, respectively. In another modeling paradigm, Ying and Miyagi (2001) and Clark and Watling (2002), respectively, developed optimization-based sensitivity analysis methods for logit-based and probit-based stochastic user equilibrium networks. The network equilibrium implied by our proposed problem is a combination of deterministic user equilibrium (for route and parking choices) and stochastic user equilibrium (for destination choice). A combination of these analytical methods can be used to conduct sensitivity analyses in our case for continuous system parameters such as link capacity and other link attributes and perturbations of these continuous parameters typically result in continuous changes of the network equilibrium state.

The driving range limit, which is a continuous parameter itself, however, does not cause a continuous change to network equilibrium flows. The underlying reason is that the distance constraint is imposed on path lengths and individual path lengths are discrete, fixed values in any given finite set of paths. As a result, the change of equilibrium network flows will be a discontinuous function of the driving range limit and hence the above sensitivity analysis methods are not appropriate for the case of the driving range limit. Let us use an example problem below to illustrate this.

The illustrative example uses a toy network with 4 nodes, 5 links, 1 origin and 2 destinations, as shown in Fig. 2. Node 1 is the origin node and nodes 3 and 4 are the destination nodes. The two terms in the parenthesis beside each link are the link travel time and link length, respectively, where x is the link flow rate. The demand departing from origin node 1 is 12. The scale parameter of the logit model for destination choice is set as $\gamma_e = 0.01$. For simplicity, we assume that all travel demands are BEVs, the travel choices of which are subject to the driving distance limit, and neither parking restriction nor parking time and cost are considered. Figure 3 shows the variation of the network flows and cost over a full spectrum of the driving distance limit, say $D \in (5, +\infty)$. It is noted that this example problem does not have any feasible solution when $D < 5$.

It can be seen clearly from Fig. 3 that a piecewise relationship exists between the link and O-D flows and the distance limit as well as between the total network cost and the distance limit. These numerical relationships are neither continuous nor monotone. If a change of the distance limit does not increase or reduce the number of used paths (and the number of O-D pairs) in the equilibrium solution set, the network flow pattern remains fixed; otherwise, the network flow pattern will in general make a saltation. In the illustrative example shown above, the key distance limit values causing the saltation of the network flow pattern include $D = 5$, 8, 10 and 13. For example, when the driving range limit is ≥ 8, both destination nodes 3 and 4 are reachable; when it is < 8, all travel demand goes to destination 3 only.

Because of the reason we discussed above, we resort to numerical evaluations to analyze the impacts of the driving range limit on network flows, which are simply a collection of the equilibrium flow results from repeatedly solving the network equilibrium problem for a prespecified set of selected parameter values. The next section depicts a partial linearization solution method for the network equilibrium problem, while the details of the numerical analysis results from implementing the solution method are then presented in the section of "Numerical analysis".

Fig. 2 An illustrative example

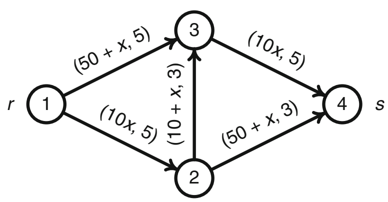
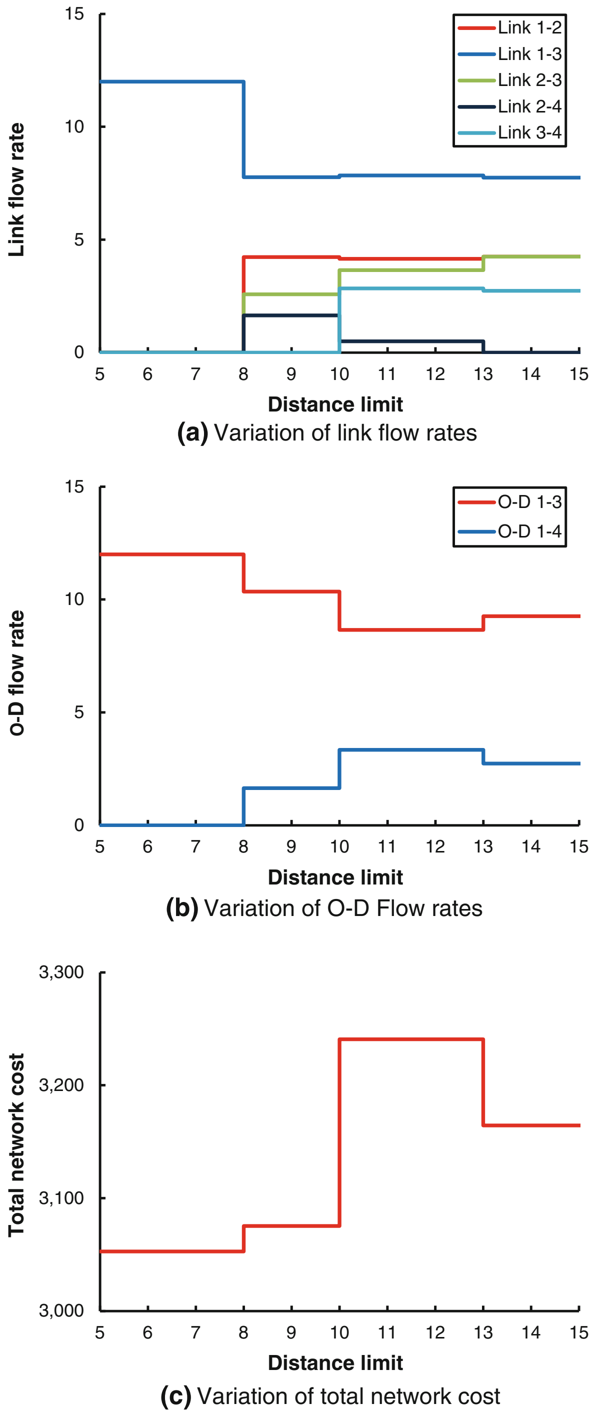


Fig. 3 Illustrative relationships between the network flows and cost and the driving range limit



Solution approach

A number of solution methods can be adapted or modified for solving the proposed distance-constrained, combined network equilibrium problem for mixed GV and BEV flows under the partial linearization framework by Evans (1976). The partial linearization strategy was originally developed for the combined trip distribution and traffic assignment problem, by which a linearized traffic assignment subproblem and a trip distribution subproblem are solved sequentially and separately at each iteration. For the linearized traffic assignment subproblem, the core procedure (i.e., the traffic loading procedure) of a few well-known solution algorithms can be adopted for its solutions, including the linear approximation (or Frank-Wolfe) algorithm and its variants (LeBlanc et al. 1975, 1985; Florian et al. 1987), convex simplex algorithm (Nguyen 1974), simplicial decomposition algorithm (Larsson and Patriksson 1992), origin-based algorithm (Bar-Gera 2002; Nie 2010), Dial's algorithm (Dial 2006), and gradient projection algorithm (Jayakrishnan et al. 1993; Chen et al. 1999, Florian et al. 2009). Solving our problem under the partial linearization framework follows a similar algorithmic process, in which the linearized traffic assignment subproblem can be solved by one of the traffic loading procedure and the parking split and trip distribution problems can be analytically solved via their complementarity system of equations and logit-based probability functions, respectively (due to the small size of the choice sets associated with the parking and destination choice problems).

In our implementation, the traffic loading procedure of the linear approximation algorithm, which results in an all-or-nothing assignment, is adapted for solving the linearized traffic assignment subproblem, given the ease of its implementation. The possibility of applying more efficient algorithms for the linearized traffic assignment problem should be discovered in further research; our focus in this paper is more on the model and result analysis rather than the solution efficiency and computational performance. In particular, the traffic loading procedure collapses to assigning all trips of GVs between an O-D pair to its minimum cost path and all trips of BEVs to its distance-constrained minimum cost path. As shown in Jiang et al. (2012), the distance-constrained minimum cost problem may be solved in an efficient manner by a hybrid preprocessing and label-setting algorithm developed by Dumitrescu and Boland (2003).

The algorithmic procedure of implementing the partial linearization method can be detailed as follows.

Step 1: Select an initial feasible solution $(x_{a,\mathrm{g}}, x_{a,\mathrm{e}}, R^{s,n}, R^{s,e}, q_{\mathrm{g}}^{rs}, q_{\mathrm{e}}^{rs})$.

Step 2: For each roadway link a, compute the generalized costs $c'_{a,\mathrm{g}}$ and $c'_{a,\mathrm{e}}$; for each parking link, compute the parking cost $\rho t^{s,n} + c^{s,n}$ or $\rho t^{s,e} + c^{s,e}$.

Step 3: For each O-D pair (r, s), determine the minimum cost path for GVs, which is the path with the minimum $p_{k,\mathrm{g}}^{rs}$ value, and set $\pi_{\mathrm{g}}^{rs} = \min_k\left(p_{k,\mathrm{g}}^{rs}\right) + \rho t^{s,n} + c^{s,n}$; determine the distance-constrained minimum cost path for BEVs, which is the path correspond to $\min_k\left\{p_{k,\mathrm{e}}^{rs} : l_k^{rs} \le D\right\}$, and set $\pi_{\mathrm{e}}^{rs} = \min_k\left\{p_{k,\mathrm{e}}^{rs} : l_k^{rs} \le D\right\} + \min\{\rho t^{s,n} + c^{s,n}, \rho t^{s,e} + c^{s,e}\}$. If no path between some O-D pair (r, s) satisfies the distance constraint, set $\pi_{\mathrm{e}}^{rs} = M$, where M is a very large number.

Step 4: Find a new set of GV and BEV demand rates $\hat{q}_{\mathrm{g}}^{rs}$ and $\hat{q}_{\mathrm{e}}^{rs}$ for each origin node r:

$$\hat{q}_{\mathrm{g}}^{rs} = O_{\mathrm{g}}^{r} \frac{\exp\left[-\gamma_{\mathrm{g}}\pi_{\mathrm{g}}^{rs}\right]}{\sum_s \exp\left[-\gamma_{\mathrm{g}}\pi_{\mathrm{g}}^{rs}\right]}$$

$$\hat{q}_{\mathrm{e}}^{rs} = O_{\mathrm{e}}^{r} \frac{\exp\left[-\gamma_{\mathrm{e}}\pi_{\mathrm{e}}^{rs}\right]}{\sum_s \exp\left[-\gamma_{\mathrm{e}}\pi_{\mathrm{e}}^{rs}\right]}$$

where π_{g}^{rs} and π_{e}^{rs} are the minimum composite path cost for GV and BEV drivers traveling between O-D pair r-s, respectively, as we defined earlier.

Step 5: Assign $\hat{q}_{\mathrm{g}}^{rs}$ to the minimum cost path obtained in Step 3 and assign $\hat{q}_{\mathrm{g}}^{s} = \sum_r \hat{q}_{\mathrm{g}}^{rs}$ to the parking link (s,n) for GVs; assign $\hat{q}_{\mathrm{e}}^{rs}$ to the distance-constrained minimum cost path and assign $\hat{q}_{\mathrm{e}}^{s} = \sum_r \hat{q}_{\mathrm{e}}^{rs}$ to one or both of the parking links (s,n) and (s,e), where the parking split result for $\hat{q}_{\mathrm{e}}^{s}$ between the ordinary and special parking facilities at destination s, $R^{s,n}$ and $R^{s,e}$, is the optimal solution of the following trivial traffic assignment problem in a three-node, two-link subnetwork that represents the parking system at destination s:

$$\min z\left(x_{\mathrm{e}}^{s,n}, x_{\mathrm{e}}^{s,e}\right) = \rho \int_0^{R^{s,n}} t^{s,n}(v)dv + \rho \int_0^{R^{s,e}} t^{s,e}(v)dv + R^{s,n}c^{s,n} + R^{s,e}c^{s,e}$$

subject to

$$x_{\mathrm{e}}^{s,n} + x_{\mathrm{e}}^{s,e} = \hat{q}_{\mathrm{e}}^{s} \quad \forall s$$

where

$$R^{s,n} = \hat{q}_{\mathrm{g}}^{s} + x_{\mathrm{e}}^{s,n} \quad \forall s$$

$$R^{s,e} = x_{\mathrm{e}}^{s,e} \quad \forall s$$

where $x_{\mathrm{e}}^{s,n}$ and $x_{\mathrm{e}}^{s,e}$ are the arrival flow rates of BEVs at destination s entering the ordinary parking facility and the special parking facility, respectively. Given the above traffic assignment and parking split results, calculate the auxiliary link flow pattern: $\hat{x}_{a,\mathrm{g}}$, $\hat{x}_{a,\mathrm{e}}$ and $\hat{x}_a$ on roadway links and $\hat{R}^{s,n}$ and $\hat{R}^{s,e}$ on parking links.

Step 6: Find the optimal step size θ^* by solving the following one-dimensional optimization problem:

$$\begin{aligned}\min z(\theta) = &\sum_{rs}\left\{\frac{1}{\gamma_{\mathrm{g}}}\overline{q}_{\mathrm{g}}^{rs}\left[\ln\left(\overline{q}_{\mathrm{g}}^{rs}\right) - 1\right] + \frac{1}{\gamma_{\mathrm{e}}}\overline{q}_{\mathrm{e}}^{rs}\left[\ln\left(\overline{q}_{\mathrm{e}}^{rs}\right) - 1\right]\right\} \\ &+ \rho\sum_a \int_0^{\overline{x}_a} t_a(\omega)\mathrm{d}\omega + \sum_a\left(\overline{x}_{a,\mathrm{g}}c_{\mathrm{g}}d_a + \overline{x}_{a,\mathrm{e}}c_{\mathrm{e}}d_a\right) \\ &+ \rho\sum_s\left(\int_0^{\overline{R}^{s,n}} t^{s,n}(v)\mathrm{d}v + \int_0^{\overline{R}^{s,e}} t^{s,e}(v)\mathrm{d}v\right) \\ &+ \sum_s\left(\overline{R}^{s,n}c^{s,n} + \overline{R}^{s,e}c^{s,e}\right)\end{aligned}$$

subject to

$$0 \leq \theta \leq 1$$

where, for any O-D pair *r*-*s*,

$$\overline{q}_{\mathrm{g}}^{rs} = q_{\mathrm{g}}^{rs} + \theta\left(\hat{q}_{\mathrm{g}}^{rs} - q_{\mathrm{g}}^{rs}\right) \quad \forall r, s$$

$$\overline{q}_{\mathrm{e}}^{rs} = q_{\mathrm{e}}^{rs} + \theta\left(\hat{q}_{\mathrm{e}}^{rs} - q_{\mathrm{e}}^{rs}\right) \quad \forall r, s$$

for any destination *s*,

$$\overline{R}^{s,n} = R^{s,n} + \theta(\hat{R}^{s,n} - R^{s,n}) \quad \forall s$$

$$\overline{R}^{s,e} = R^{s,e} + \theta(\hat{R}^{s,e} - R^{s,e}) \quad \forall s$$

and for any link *a*,

$$\overline{x}_{a,\mathrm{g}} = x_{a,\mathrm{g}} + \theta\left(\hat{x}_{a,\mathrm{g}} - x_{a,\mathrm{g}}\right) \quad \forall a$$

$$\overline{x}_{a,\mathrm{e}} = x_{a,\mathrm{e}} + \theta\left(\hat{x}_{a,e} - x_{a,e}\right) \quad \forall a$$

$$\overline{x}_{a} = \overline{x}_{a,\mathrm{g}} + \overline{x}_{a,\mathrm{e}} \quad \forall a$$

The updated solution $(x_{a,\mathrm{g}}, x_{a,\mathrm{e}}, R^{s,n}, R^{s,e}, q_{\mathrm{g}}^{rs}, q_{\mathrm{e}}^{rs})$ can then be calculated by the above functions with the optimal step size θ^*.

Step 7: Check the stopping criterion. If $\Delta < \varepsilon$, stop; otherwise return to step 2, where Δ is the average gap of link flows and parking flows between consecutive iterations and ε is a predetermined convergence criterion for Δ.

Numerical analysis

Our adapted version of the partial linearization method was coded in C++. Implementing this iterative solution method requires solving a few linear, nonlinear or combinatorial optimization subproblems at each iteration. The most computationally intensive part is the solution of the distance-constrained minimum cost problem (see Step 3), which poses NP-hard complexity. We adopted Dumitrescu and Boland's (2003) modified label-setting algorithm with preprocessing for its solution. As for the basic minimum cost problem (see Step 3), Dijkstra's (1959) classic label-setting algorithm is used in our implementation. Another optimization subprocess in implementing the Evans method is to find the optimal step size θ at each iteration (see Step 5). We suggested the bisection search for this one-dimensional nonlinear optimization problem. In all the experiments, the convergence criterion for the flow difference between consecutive iterations is set as $\varepsilon = 0.001$.

This section contains a presentation of the numerical analysis results we obtained from applying the partial linearization method for solving the combined equilibrium problem in a couple of synthetic and realistic example networks. The purpose of this numerical analysis is twofold: (1) to illustrate and justify the correctness of the modeling and solution methods for the distance-constrained, combined network equilibrium problem; (2) to assess the impacts on the network performance from different BEV battery capacities or driving range limits under different network scenarios, such as different BEV penetration rates. The driving range limit is a key specification of BEVs and a determinant factor that stimulates or discourages the purchase willingness of potential BEV users.

For both the synthetic and realistic networks, we use the following link travel time function and parking access and search time functions to describe the supply performance on network links and parking facilities. The link travel time function employs the form of the Bureau of Public Roads (BPR) function:

$$t_a = t_a(x_a) = t_{a,0}\left(1 + \alpha_a\left(\frac{x_a}{m_a}\right)^{\beta_a}\right) \quad \forall a$$

where $t_{a,0}$ is the free-flow travel time of link a, m_a is the link capacity or maximum flow rate, and α_a and β_a are function parameters. Similarly, we assume that the parking access and search time function has a BPR functional form as well,

$$t^{s,n}(R^{s,n}) = t_0^{s,n} + \alpha_{s,n}\left(\frac{R^{s,n}}{m^{s,n}}\right)^{\beta_{s,n}} \quad \forall s$$

$$t^{s,e}(R^{s,e}) = t_0^{s,e} + \alpha_{s,e}\left(\frac{R^{s,e}}{m^{s,e}}\right)^{\beta_{s,e}} \quad \forall s$$

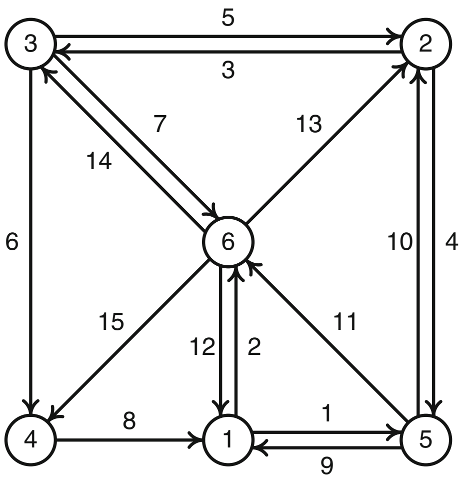
where $t_0^{s,n}$ (or $t_0^{s,e}$) is the free-flow parking access and search time for the ordinary parking facility (or the special parking facility), $m^{s,n}$ (or $m^{s,e}$) is the capacity of the ordinary parking facility (or the special parking facility), and $\alpha_{s,n}$ and $\beta_{s,n}$ (or $\alpha_{s,\mathrm{e}}$ and $\beta_{s,\mathrm{e}}$) are function parameters for the ordinary parking facility (or the special parking facility). These performance function parameters reflect the non-measurable performance specifications of the roadway and parking infrastructures and determination of these parameter values is critical to the success of evaluating the network performance. Systematically calibrating parameters over the whole network is often very challenging. In the two example problems given below, we assume that all the required function parameter values are known a priori.

The Lam-Huang network

To look into the traffic and parking flow patterns on the link, O-D and destination levels given different driving range limits, a small network shown in Fig. 4 is used as an illustrative example. This network was originally used by Lam and Huang (1992) to study a combined trip distribution and traffic assignment problem. The network includes 15 links, 6 nodes and 10 O-D pairs. Nodes 1, 2 and 3 represent origin zones; nodes 1, 2, 4 and 5 represent destination zones.

The following parameter values are applied to the Lam-Huang network. The scale parameters of the logit model for destination choice: $\gamma_{\mathrm{g}} = 0.0039$ and $\gamma_{\mathrm{e}} = 0.0057$; the parameters of the link travel time function: $\alpha_{\mathrm{a}} = 0.15$ and $\beta_{\mathrm{a}} = 4$; the value of time: $\rho = 4$; the unit operation cost: $c_{\mathrm{g}} = 3.85$ and $c_{\mathrm{e}} = 1.28$; the

Fig. 4 The Lam-Huang network



parameters of the parking access and search time function for all destinations: $\alpha_{s,n} = \alpha_{s,\mathrm{e}} = 0.1$ and $\beta_{s,n} = \beta_{s,\mathrm{e}} = 3$; and, finally, for simplicity, the parking cost: $c^{s,n} = 0$ and $c^{s,e} = 0$. In addition, the supply and demand data sets for modeling the Lam-Huang network include the following. The link free-flow travel times are: $\left(t_{1,0}, t_{2,0}, \ldots, t_{15,0}\right) = (22, 12, 25, 38, 70, 55, 20, 20, 35, 25, 18, 15, 18, 14, 23)$; the link distances are: $(d_1, d_2, \ldots, d_{15}) = (3.8, 1.2, 2.0, 3.9, 5.6, 4.2, 2.8, 2.8, 3.9, 3.1, 1.5, 2.4, 2.3, 1.5, 1.9)$; the link capacity is assumed to be the same on all links: $m_a = 335$; the parking free-flow times are: $\left(c_0^{1,n}, c_0^{2,n}, c_0^{4,n}, c_0^{5,n}\right) = (5, 4, 5, 4)$ and $\left(c_0^{1,e}, c_0^{2,e}, c_0^{4,e}, c_0^{5,e}\right) = (2, 3, 2, 3)$; the capacities of the ordinary and special parking facilities are also assumed to be the same with all destinations: $m^{s,n} = 500$ and $m^{s,e} = 200$, respectively; the departing demand rates at the origin nodes: $\left(O_{\mathrm{g}}^1, O_{\mathrm{g}}^2, O_{\mathrm{g}}^3\right) = (250, 120, 430)$ and $\left(O_{\mathrm{e}}^1, O_{\mathrm{e}}^2, O_{\mathrm{e}}^3\right) = (290, 160, 260)$. The departing demand pattern indicates the penetration rate of BEVs ranging from 37.7 to 57.1 % across origins. Finally, note that other parameters and data listed above are directly copied from Lam and Huang (1992).

It has been emphasized that the distinguishing feature of our model, different from previous models of this type, is the introduction of a driving range constraint. It is of our particular interest to assess how the constraint with different upper bounds (i.e., distance limits) affects the computation results and reshapes the network flow patterns across different network components.

The relationship between the system performance and the driving range limit is first examined. Figure 5 depicts the variations of the objective function value and the total system cost (i.e., the sum of all driving and parking times and monetary cost) over a range of distance limits. From the optimization objective, it is well known that the objective function value of any minimization problem turns higher when any of its constraint becomes looser. This phenomenon is clearly observed in the Fig. 5a. More specifically, the objective function value drastically decreases in the section of the distance limit from 5 to 8, while it only slightly changes when the distance limit is lower than 5 or higher than 8. Recall that the distance limit in general causes a discontinuous change of the network equilibrium state because of the discrete nature of path lengths. Similarly, the variation of the total system cost shows a similar 3-section pattern, in which its value changes significantly in the section of the distance limit between 5 and 8, as shown in Fig. 5b. However, different from the objective function value, the total system cost is not monotone over the given range of distance limits. This result clearly shows the possible outcome similar to the Braess paradox, in which increasing the distance limit does not necessarily increase or decrease the total system cost.

We further proceed to examine how the network flow patterns shift due to the change of the distance limit on the network links, O-D pairs and parking facilities. Given the small size of the Lam-Huang network, we can conveniently exhaust the results for all individual network components. The results for the network links, O-D pairs and parking facilities are given in Figs. 6, 7 and 8, respectively. In each of these figures, three representative distance limit values, namely, $D = 4$, 6, and 10, which are picked up from the 3-distance limit sections shown in Fig. 5a, respectively, and the case of no distance limit are used. The latter is used as the base case for the comparison purpose. Again, in all these cases, the penetration rate of BEVs is fixed, ranging from 37.7 to 57.1 %, across different origin nodes.

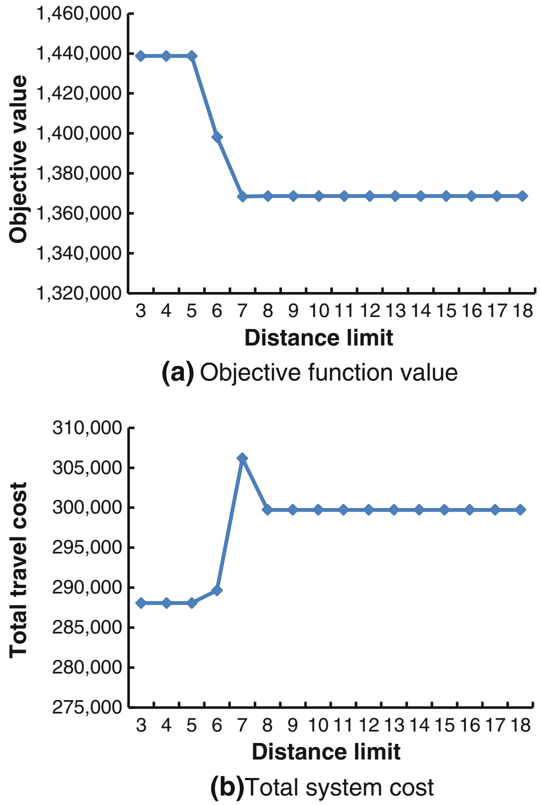


Fig. 5 Variation of system performance measures over a range of distance limits

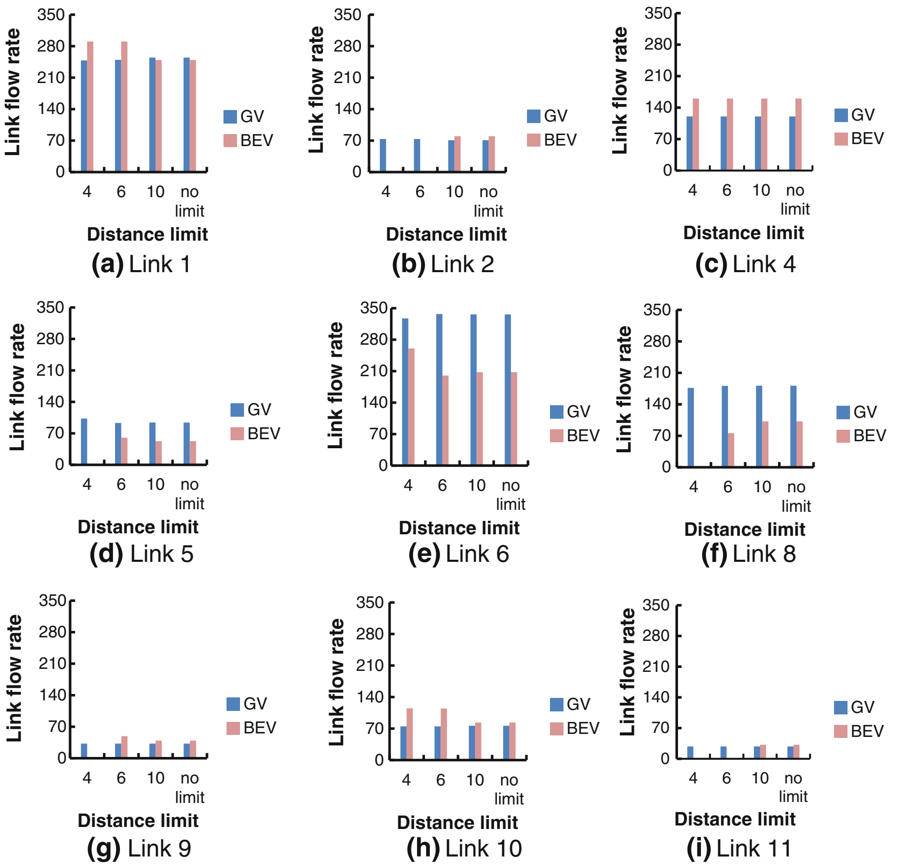


Fig. 6 Variation of individual link flow rates with different distance limits (links with zero flows are omitted in this figure)

It is evident from Fig. 6 that BEV link flows change much more significantly, due to the change of the distance limit, than GV link flows. The result is intuitively understandable: travel choices of BEVs are directly restricted by the distance limit. For example, when $D = 4$, which represents the tightest distance limit in Fig. 6, there are only four paths available for BEVs, namely, path 1-5-2 between O-D pair 1-2, path 1-5 between O-D pair 1-5, path 2-5 between O-D pair 2-5 and path 3-4 between O-D pair 3-4. As a result of this limited feasible path set for BEVs, only links 1, 4, 6 and 10 carry BEV flows. With a higher distance limit, more paths are eligible for BEVs and hence more links carry BEV flows.

The O-D flow changes show a similar pattern in comparing the GV and BEV flows. In particular, the trip distribution of BEVs is more strongly affected by the changed distance limit than that of GVs. For example, when $D = 4$, BEVs can only travel between O-D pairs (1, 2), (1, 5), (2, 5) and (3, 4), as shown in Fig. 7. Since no BEV heads to destination 1, the parking demand for the special parking facility at destination 1 is zero, as shown in Fig. 8. From this figure, it is observed that parking

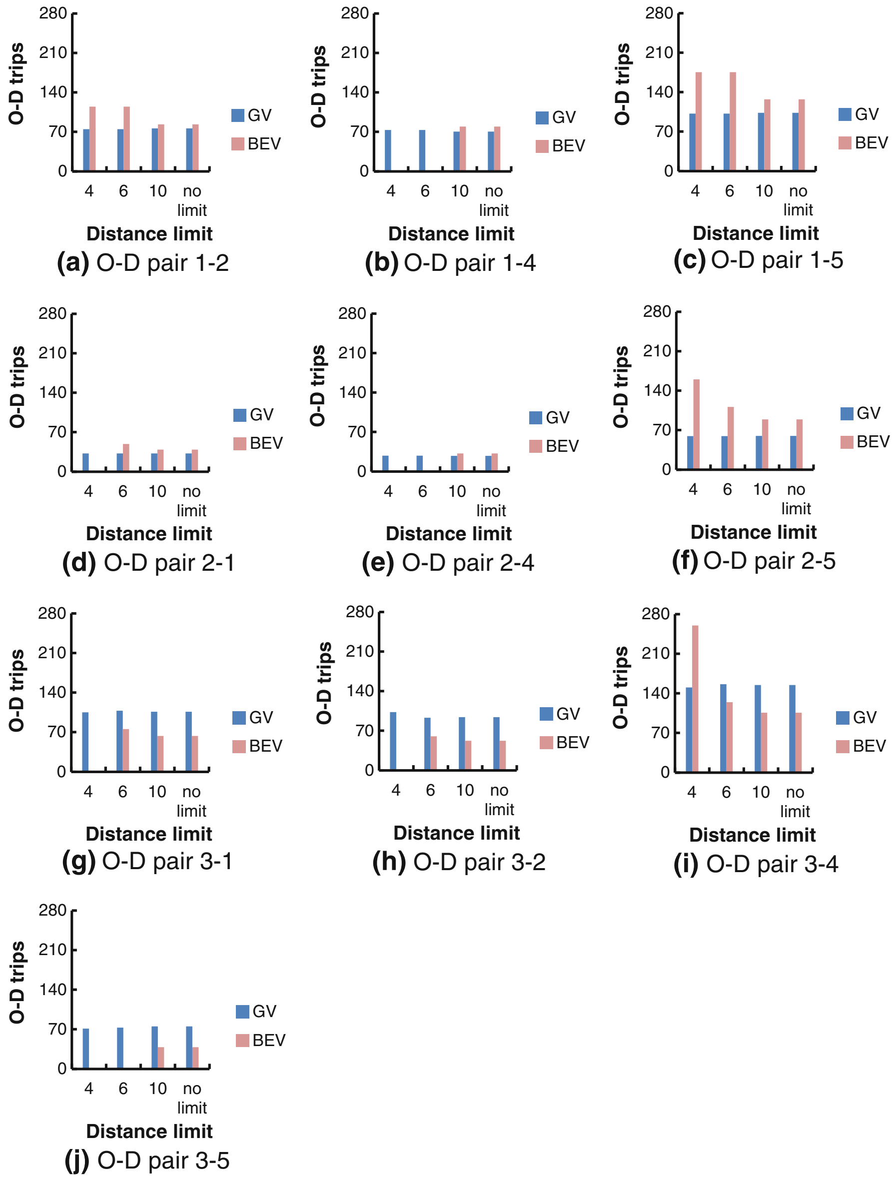


Fig. 7 Variation of O-D trip rates with different distance limits

flows entering the special parking facilities are typically more fluctuant to the change of the distance limit than the ordinary parking facilities.

In all the above figures (i.e., Figs. 6, 7 and 8), it is found that the flow results for the cases of $D = 10$ and no distance limit are the same. A closer look into the problem reveals the reason that the distance constraint is in fact not binding at the optimal solution of this example problem when the distance limit D is higher than 7.

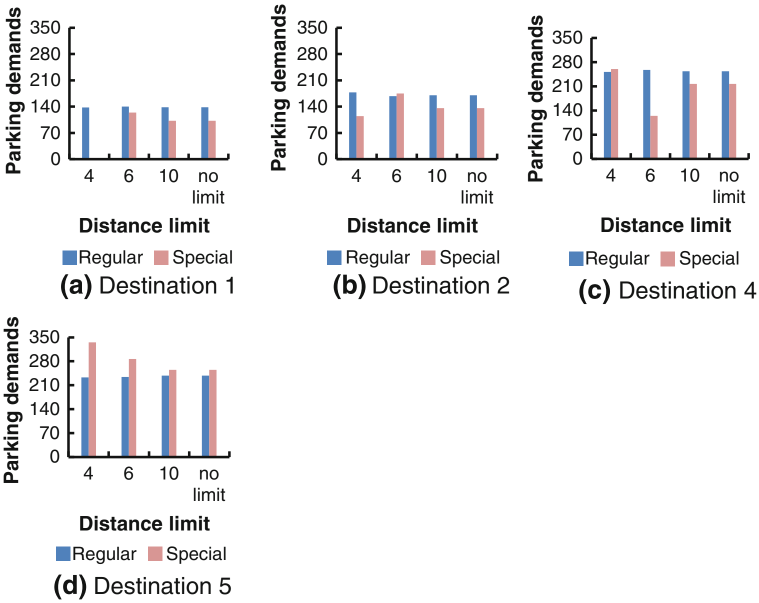


Fig. 8 Variation of parking demand rates with different distance limits

The Anaheim network

In addition to the synthetic results about individual network components from the above toy network, we are more interested in the system-wide effects imposed by the distance constraint, under different penetration levels of GVs and BEVs and from real-world transportation networks. The Anaheim network (shown in Fig. 9) is used here to serve this purpose. This network has a typical urban grid topology and poses a medium-size problem in the context of transportation planning: 38 origin and/or destination zones, 416 nodes, and 914 links.

It is hypothesized that the total departing demand rates from all origin zones are fixed while the penetration rates of BEVs and GVs vary. The underlying assumption is that the automobile market in the urban area (e.g., Chicago, Philadelphia, and Providence in the US) is relatively stable and the total travel demand in the network reaches a saturated level, but the penetration rate of BEVs are expected to rapidly increase, for which the major part of new BEV purchases is from the replacement of old GV models. Even if the travel demand may still experience increases with the population and business growth, its increasing rate would be marginal compared to the increasing rate of the number of BEVs. If a region with fast population growth (e.g., Houston, Austin, and Phoenix in the U.S.) is modeled, the corresponding travel demand increase should be certainly taken into account. Moreover, for modeling simplicity, we set the same penetration rate across all origin zones in all scenarios of distance limits and penetration levels.

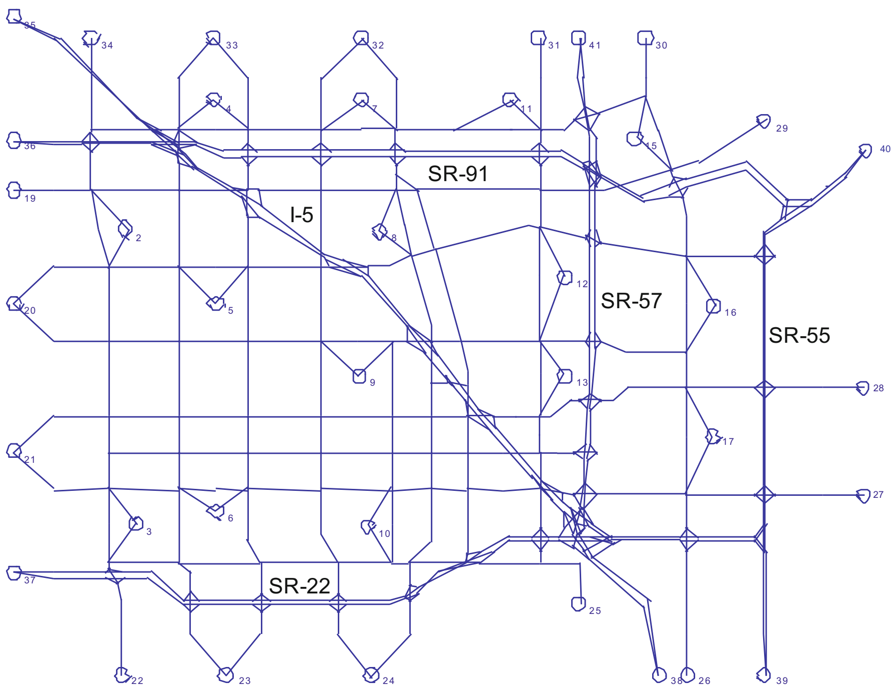


Fig. 9 The Anaheim network

All basic supply and demand data and parameter sets can be viewed from the network problem's source website,[2] so we do not present them here. Additional parameters pertaining to our specific setting are given as: value of time is $10/hour ($0.16/min), the unit operation costs of GVs and BEVs are $0.16/mile and $0.04/mile, respectively, the parking fees of the regular and special parking facilities at all destinations are $5 and $3, respectively, the capacities of the regular and special parking facilities at all destinations are 500 and 300 vehicles, respectively, and the parameters of the parking access and search time function are $\alpha_{s,n} = \alpha_{s,\mathrm{e}} = 0.024$ and $\beta_{s,n} = \beta_{s,\mathrm{e}} = 4$.

Firstly, we checked the network influence from different distance limits ($D = 5$, 10 and 15 miles[3]) and BEV penetration rates (10, 50 and 90 %) on the computational efficiency of the algorithm. The results shown in Fig. 10 reveal that when the penetration rate of BEVs is relatively high, the impact of the distance constraint on the convergence performance is significant and the significance becomes more obvious when the distance constraint is tighter (i.e., the distance limit is smaller); when the BEV penetration rate is relatively low, the impact of the distance limit on the convergence performance may be negligible. In overall, under the same distance limit, the higher the

[2] A set of transportation network test problems maintained by H. Bar-Gera can be accessed at http://www.bgu.ac.il/~bargera/tntp/.

[3] Most of BEV models in the market have a much higher driving range limit than the values listed here. Given that the geographic size of the Anaheim network is relatively small, these driving range limit values are deliberately hypothesized for the illustration purpose only; otherwise, the impacts of the driving range limit on network flow shifts may not be apparent.

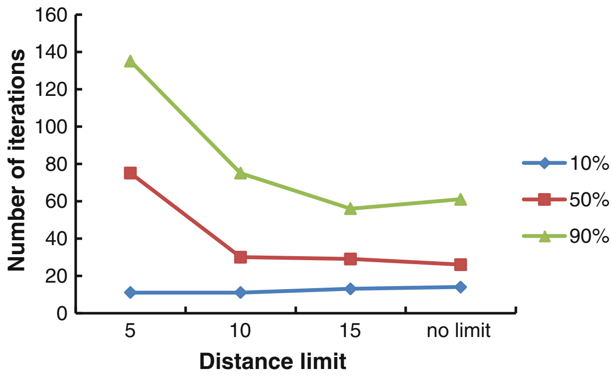


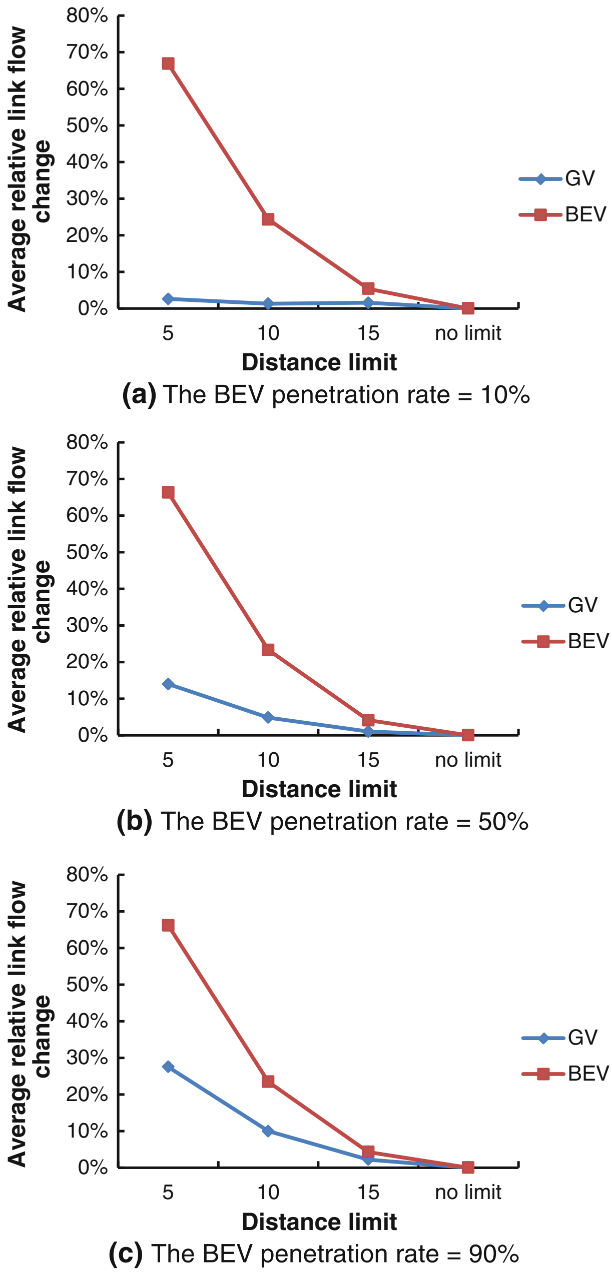
Fig. 10 The number of iterations with different BEV penetration rates and distance limits

penetration rate is, the larger the number of iterations is required for achieving the same convergence level. This result is also true even when there is no distance limit imposed on BEVs. Under this condition, the major reason for the extra computational cost is from the parking availability restriction, in that BEVs can choose between two parking facilities while GVs can only choose ordinary parking lots. This phenomenon is also proved by an alternative experiment result from limiting the parking choice of BEV drivers to only special parking facilities. In that case, 41 iterations are required for solution convergence, which is smaller than 75 iterations, the number of iterations required for solution convergence for the case of BEVs choosing between two types of parking facilities, given that the distance limit is 5 and penetration rate is 50 %.

Next, we evaluated the network-wide impacts of different range limits and BEV penetration rates on the average relative changes of O-D, link, and parking flows of both GVs and BEVs. The equilibrium flow results without any distance limit are used as the base case. The formula for calculating average relative changes of O-D, link, and parking flows are given as follows, if, for example, the quantity of interest here is the average relative change of the GV link flow and the distance limit is D_m:

$$\frac{\sum_a \left| x_{a,g}^{D=D_m} - x_{a,g}^{D=m} \right|}{\sum_a x_{a,g}^{D=\infty}}$$

Figure 11 below records the results of the average relative changes of link flows given different distance limits. All of the plots in Fig. 11 under different penetration rates of BEVs (and GVs) show a similar link flow variation pattern: BEV link flows are much more significantly influenced by the distance limit than GV link flows. Moreover, the smaller the distance limit is, the more apparent the change of BEV link flows becomes. This reflects the phenomenon that the distance limit directly changes the destination and route choice behaviors of BEV flows but influences in an indirect way the travel behaviors of GV flows via the change of BEV flows. We also notice that with the increase of the penetration rate of BEVs, GV links flows are more likely influenced by the distance limit. This is a natural response to the increasing change of BEV flows.



(a) The BEV penetration rate = 10%

(b) The BEV penetration rate = 50%

(c) The BEV penetration rate = 90%

Fig. 11 Variation of average relative changes of link flows with different penetration rates and distance limits

Figure 12 summarizes the result of the average relative change of parking demand rates in terms of the distance limit. All the cases show that the impacts of the distance limit on both the ordinary and special parking demand rates increase as the distance limit reduces, except the ordinary parking facilities under the low penetration rate (i.e., 10 %). This is mainly because when the distance constraint turns tighter, fewer destinations are reachable by BEVs, which shifts the BEV parking demands to concentrate to those parking facilities at reachable destinations. However, across different penetration rates of BEVs, the impacts on the ordinary and special parking facilities show a different tendency. With the increase of the penetration rate of BEVs (which also means the decrease of the penetration rate of GVs, since we assume that the total travel demand is fixed), the average relative parking demand changes with the ordinary and special parking facilities go down and up, respectively. This is simply the equilibrium result from the setting that the ordinary parking facilities can accommodate both GVs and BEVs while the special parking facilities serve BEVs only.

Finally, the same comparison scheme is applied to the O-D flow patterns in the Anaheim network, which leads to the result shown in Fig. 13. It clearly shows that the changes of BEV O-D flows are more sensitive to the change of the distance limit; with the decrease of the distance limit, the change of BEV O-D flows becomes increasingly apparent. This is not surprising, since when the distance constraint becomes tighter, more destinations are not accessible by BEVs and BEV flows are concentrated to a fewer number of destinations. Comparing the three plots under different penetration rates of BEVs, we notice that the impact of the distance limit on GV O-D flows also increases with the increase of the penetration rate of BEVs. This can also be characterized as a follow-up effect due to the BEV O-D flow pattern changes as we have discussed for link flows.

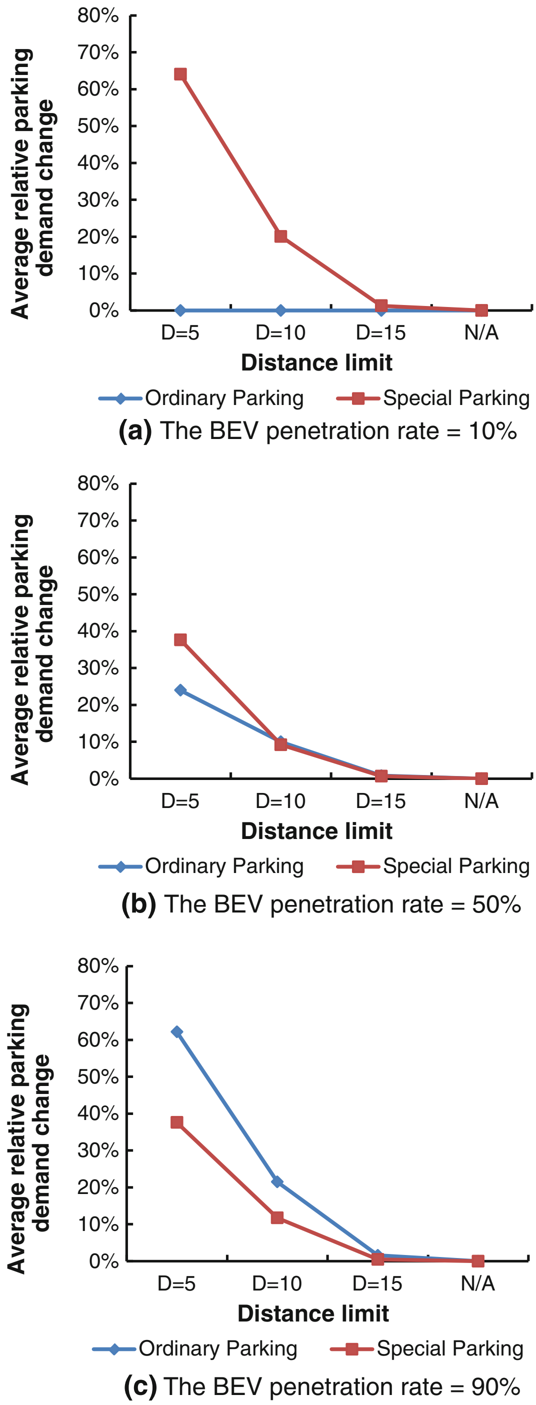
In overall, these comparison results justify the hypothesis that the network flow shifts caused by the driving range limit are significant and complex and these changes become more apparent as the distance constraint goes tighter and the penetration rate goes higher. Based on these observations, we can conclude that when the penetration level of BEVs in the market climbs to a certain level, the existing network equilibrium and travel demand modeling tools must be revised to accommodate such changes. The model constructed in this paper provides an initial attempt, but more modeling efforts need to be made to support other parts of the existing travel demand modeling framework.

In addition, it should be noted that the numerical analysis results shown above merely provide approximate relationships between the network flows and driving range limit, which are estimated by selecting some representative limit points. To accurately depict the relationship profiles (as shown in Fig. 3), we need to first identify those key driving range limits that cause the network flow pattern to shift and estimate network flows at these key limits. A straightforward approach to identify the key driving range limits over the whole network is to apply a *k*-shortest path algorithm to each O-D pair.

Conclusions and future directions

In summary, this paper presents a special combined network equilibrium model with a driving range limit and multiple types of vehicles for responding to the recent

Fig. 12 Variation of average relative changes of parking demands with different penetration rates and distance limits



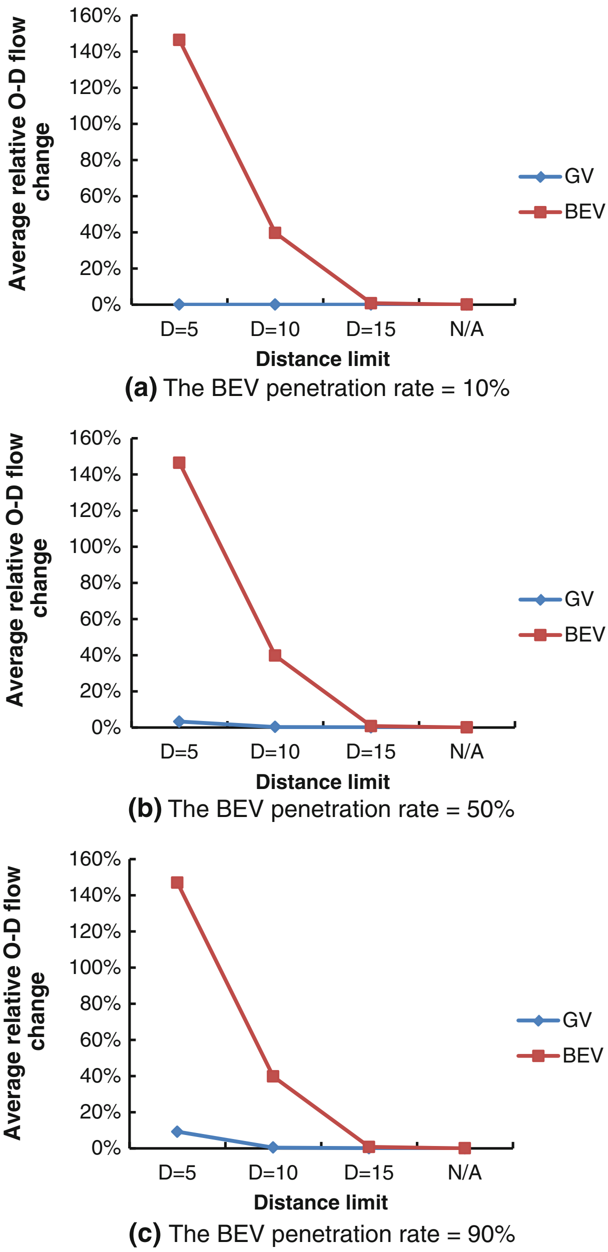


Fig. 13 Variation of average relative changes of O-D flows with different penetration rates and distance limits

research calls for new travel demand modeling and network analysis tools for the coming era of electric vehicles. The model is constructed in the simplest form of its type and our research focus is given to understand how the network flow pattern on the O-D pair, parking facility, and network link levels are reshaped by the introduction of BEVs into the market and how BEV and GV traffic flows interact and compete with each other in a combined urban traffic routing-parking system. The added driving range constraint on path lengths of BEVs imposes a new restriction on all travel choice behaviors and raises alternative algorithmic complexities. As we showed in our previous research and this paper, solving the distance-constrained minimum cost problem is required in quantifying the route choice behavior of individual BEV drivers, for instance.

The combined network equilibrium problem has been solved by an adapted partial linearization solution method, in which the linearized traffic assignment problem is tackled by the linear approximation procedure, the trip distribution subproblem is evaluated by the logit-based probability functions, and the parking split result is directly obtained by solving the complementarity system of equations. The computational results from applying the solution method for the two example network problems with small and medium sizes clearly illustrate the significant impacts on the network flow patterns imposed by the alternative travel cost composition and travel distance limit associated with BEVs. This finding, on one hand, indicates the need of using updated travel behavior and travel demand modeling tools for accommodating new travel flexibilities and restrictions brought by the future massive adoption of electric vehicles (and other alternative-fuel vehicles); on the other hand, it implies possible new opportunities for transportation planners and operators to reshape travel and activity patterns and reduce urban congestions and pollutants on both the planning and operational levels.

In overall, the modeling and solution methods presented in this paper provide us with a fundamental tool to understand transportation network changes due to the destination, route and parking choice adjustments with BEV drivers. Starting from this fundamental tool, we may extend its functionalities to capture more realistic transportation network phenomena by adding extra modeling components, recasting supply and demand assumptions, and employing more advanced modeling paradigms.

It is very likely that the automobile market will be comprised of GVs, PHEVs and BEVs (as well as other types of electric vehicles) simultaneously for a long period in the predictable future. Unlike BEVs, PHEVs are not subject to the distance limit and are becoming a very attractive transportation mode to consumers, at least in the transition period between the gasoline era and electricity era. From the modeling perspective, this type of vehicles poses a different distance-dependent energy efficiency curve and operating cost structure. More specifically, a PHEV can be driven in two different operating modes, defined by the net effect on the battery state of charge: charge depleting and charge sustaining. A typical driving mode for a PHEV is that it is powered first by electricity only or a combination of electricity and gasoline in the charge-depleting mode; once the battery is depleted to a minimum state of charge, a PHEV uses only gasoline energy in the charge-sustaining mode. Introducing PHEVs into the already complex mixed GV-BEV traffic network adds another layer of complexity to the modeling framework presented in this paper.

In our network equilibrium analysis, we implicitly assume that all BEVs are fully charged at their origins (e.g., home garages), and ignore possible availability of commercial battery-charging or battery-swapping stations emerging in urban areas in the near future. In most transportation networks, it may take a number of years to deploy sufficient electricity-recharging infrastructures for achieving a certain level of coverage. Nevertheless, a partial coverage still offers some recharging opportunities for BEV drivers and favors their travel choice decisions. Introducing a limited set of commercial charging or swapping stations (in addition to home garages) into any BEV-contained network models will inevitably create new modeling complexities and algorithmic challenges. For example, the individual route choice behavior with BEV drivers in this case will no longer be formulated as a distance-constrained minimum-cost problem, but a minimum-cost problem with relays (Laporte and Pascoal 2011; Smith et al. 2012). Apparently, a more sophisticated network equilibrium model that accommodates such travel choice behaviors is necessary and will be very useful in evaluating relevant plans and policies of developing and maintaining electricity-recharging infrastructures.

Finally, we would like to emphasize that realistic travel and parking behaviors and network flow congestion conditions can be much better characterized by a dynamic network model. While a large number of analytical and simulation-based dynamic traffic assignment models can be used for describing network-wide traffic dynamics, a detailed parking access and search process (e.g., Thompson and Richardson 1998) should be also employed for accounting for parking dynamics. This is not only a requirement for capturing time-varying traffic network states and developing time-of-day parking policies (as well as many other time-dependent network control strategies), but also a critical prerequisite for the model potentially used for evaluating the policy and operational issues of implementing BEVs as distributed and mobile electricity storage to support coupled transportation-energy systems (Kezunovic and Waller 2011).

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