EURO J Transp Logist (2014) 3:227–243
DOI 10.1007/s13676-013-0032-2

RESEARCH PAPER

Relationship between mean and day-to-day variation in travel time in urban networks

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Received: 16 November 2012 / Accepted: 10 August 2013 / Published online: 29 August 2013
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Abstract The day-to-day reliability of transportation facilities significantly affects travel behavior. To better understand how travelers use these facilities, it is critical to understand and characterize this reliability for different facilities. Early work in this area assumed that the variance of day-to-day travel times (a measure of the inverse of reliability) increases proportionally with the mean travel time; i.e., as the mean travel time increases, travel time reliability decreases. However, recent empirical data for a single bottleneck facility and a small urban network suggest a more complex relationship that exhibits hysteresis. When this phenomenon is present, the variance in travel time is larger as the mean travel time decreases (congestion recovery) than as the mean travel time increases (congestion onset). This paper presents an elegant theoretical model to describe the variance of travel times across many days in an urban network. This formulation shows that the hysteresis behavior observed in empirical floating car data on urban networks should not be unexpected, and that it is linked to the hysteresis loops that often exist in the Macroscopic Fundamental Diagram of urban traffic. To verify the validity of this formulation, data from a micro-simulation of the City of Orlando, Florida, are

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used to derive an observed relationship with which to compare to theory. The simulated data are shown to match the theoretical predictions very well, and confirm the existence of hysteresis in the relationship between the mean and variance of travel times that is suggested by theory. These results can be used as a first step to more accurately represent travel time reliability in future models of traveler decision-making.

Keywords Hysteresis · Travel time reliability · Macroscopic fundamental diagram · Urban traffic network dynamics

Introduction

Day-to-day fluctuations in travel time can range from a minor annoyance to a significant contributor of a travelers' decision-making process. Researchers have recognized the potential impact of travel time variability on decision-making and tried to incorporate this metric into behavioral models. For example, the variability of travel time has been shown to impact, and has been incorporated into models of, both vehicle routing (Small 1999; Brownstone and Small 2005; Bates et al. 1987, 2001) and trip departure times (Polak 1987; Polak et al. 2008; Koster and Verhoef 2012). More recent work has attempted to quantify the relative value of travel time reliability (i.e., the inverse of travel time variation) as compared to the value of mean travel time. These studies all suggest that travelers value reliability almost as much as they value mean travel time; more specifically, the value of travel time reliability ranges between 0.5 and 0.8 times as much as mean travel time (Hollander 2006; Asensio and Matas 2008; Li et al. 2010).

As travel demand and choice models start to incorporate travel time variability metrics into their evaluation framework, relationships between mean (or expected) travel time and travel time variability have become increasingly important. Recent work has used empirical data to derive this relationship. For example, the SHRP2 Project "Analytical procedures for determining impacts of reliability mitigation strategies" (SHRP 2010) produced a model that suggests travel time variability increases proportionally to mean travel time. This type of increasing relationship has also been suggested and used in other studies (VanLint et al. 2008; Sirivadidurage et al. 2009; Peer et al. 2010).

However, empirical data suggest that the relationship between mean and variance of travel times might not be that simple. Consider the data shown in Fig. 1 from Fosgerau (2010) that describes the mean and variance of travel times measured for various time periods over many days at a single facility in Copenhagen. While a myopic regression might yield an increasing relationship between mean and variance of travel times, the relationship is actually more nuanced. Travel time variance clearly increases with mean travel time, reaches a peak and then declines with mean travel time, as expected. But the rate of change of travel time variance with mean travel time is very different as the mean travel time increases (i.e., as congestion begins to set in) than as the mean travel time decreases (i.e., as congestion begins to dissipate). This causes a hysteresis pattern where the variance

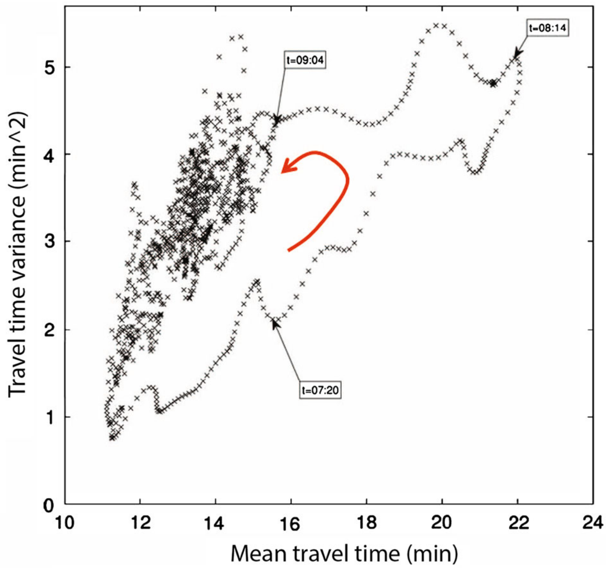

Fig. 1 Hysteresis observed on an 11-mile road section in Copenhagen (Fosgerau 2010)

of travel times measured across many days (hereafter referred to as day-to-day travel time variance) depends not only on the mean travel time, but also on the change in the mean travel time or time of day. This anti-clockwise pattern in the relationship between mean and variance of day-to-day travel times has also been observed at a network-wide level in Leeds, UK (Bates et al. 2003); see Fig. 2. Notice that travel time variation is consistently lower during periods when the mean travel time increases than when the mean travel time decreases. However, it is not known if these patterns are site specific or should be expected in general.

Fosgerau (2010) provided a theoretical explanation for this anti-clockwise hysteresis behavior at isolated bottlenecks. This study used the classic Vickrey morning commute model (Vickrey 1969) to show that anti-clockwise patterns are caused by random fluctuations in both vehicle demand to the bottleneck (i.e., arrival rates) or supply of the bottleneck (i.e., its capacity). Therefore, this work suggests that these loops should be generally expected at isolated bottlenecks. However, what is less understood is if this type of hysteresis pattern should be expected on urban networks that exhibit interactions between bottlenecks, traffic signals, multiple routes, conflicting movements and other complexities.

Recent work on the aggregate behavior of vehicles on urban networks might be able to shed some light on this phenomenon. Daganzo (2007) recently reintroduced the idea that a reproducible relationship exists between average flow and density in urban networks, which was first introduced by Godfrey (1969). This relationship has come to be known more commonly as the Macroscopic Fundamental Diagram (MFD) or the Network Fundamental Diagram (NFD). The existence of the MFD was verified by simulation (Geroliminis and Daganzo 2007), traffic flow theory (Daganzo and Geroliminis 2008) and, perhaps most importantly, empirical data

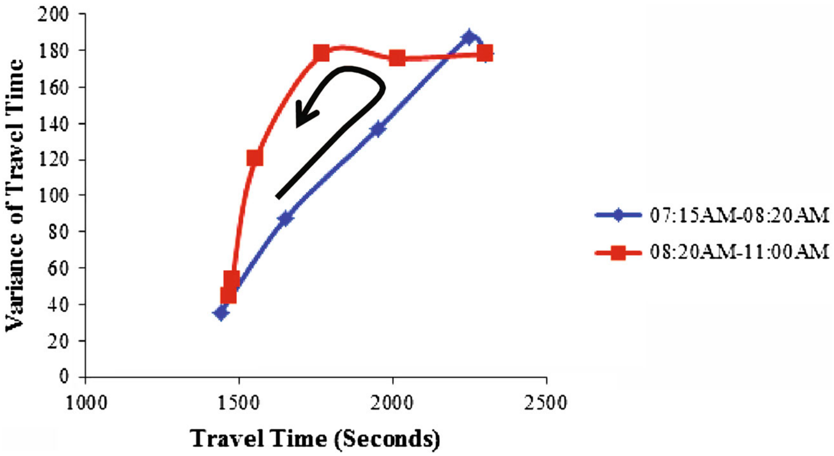

Fig. 2 Hysteresis observed on the Leeds street network (Bates et al. 2003)

(Geroliminis and Daganzo 2008). This type of model has shown to be extremely useful to understand network dynamics and devise large-scale network control strategies that can be implemented across an urban network. Some examples include Knoop et al. (2012), Haddad and Geroliminis (2012), Daganzo et al. (2012), Keyvan-Ekbatani et al. (2012) and Gayah and Daganzo (2012); note, however, that this is just an illustrative list and is not comprehensive.

Simulation (Aboudolas et al. 2010; Gayah and Dixit 2013) and empirical studies (Buisson and Ladier 2009; Geroliminis and Sun 2011; Saberi and Mahmassani 2012) have also shown hysteresis patterns exist in plots of the aggregate flow–density relationship. Average network flows are found to be consistently higher as average network density increases (during the onset of congestion) than as average network density decreases (during congestion dissipation); i.e., the flow–density relationship exhibits a clockwise hysteresis loop. A theoretical explanation of this clockwise loop was provided in Gayah and Daganzo (2011), which attributed the observed behavior to naturally occurring instabilities that exist in congested urban networks (Daganzo et al. 2011). The former work also provided theory to suggest that the clockwise patterns in the flow–density relationship should be expected in networks if drivers do not have perfect real-time travel information or cannot change routes easily to avoid localized pockets of congestion. This would occur even in the most optimistic case that the network is homogeneous and vehicles use all portions of it uniformly.

It seems likely that this phenomenon could be linked to the hysteresis pattern observed in the relationship between mean and variance of travel time. To make this connection, this paper develops a model that describes mean travel time and day-to-day travel time variance during a particular time period as a function of the average flow and the average density measured across many days. The model is then used to show that clockwise hysteresis loops in the flow–density relationship would imply anti-clockwise loops in the relationship between mean and variance of travel time.

The insights obtained from the theoretical model are verified using data from a micro-simulation of the urban network of downtown Orlando, Florida. Since clockwise hysteresis loops in the MFD are the most likely pattern, the model and its results suggest that anti-clockwise loops between mean and variance of day-to-day travel times should not be unexpected.

The rest of this paper is organized as follows. "Theoretical model" provides the theoretical model to describe day-to-day variation in travel times. "Relationship between day-to-day travel times and the MFD" discusses the connection between reproducible clockwise hysteresis loops in the MFD and day-to-day travel time variance. "Verification using simulation" uses trajectory data from a micro-simulation of a realistic urban network to verify the theoretical model and its conclusions. "Concluding remarks" provides some concluding remarks.

Theoretical model

We consider here an urban network with a recurring rush period that is divided into I discrete analysis periods. Each of these periods is indexed by $i \in \{1, 2, ..., I\}$. The rush period is assumed to occur for a time period of J days, with each day indexed by $j \in \{1, 2, ..., J\}$. We now develop analytical equations for average flow, average density, mean travel time and travel time variance.

During a particular period i on day j, the average flow and density aggregated across the entire network can be calculated using the generalized definitions provided by Edie (1965). With these definitions, the network flow is given by:

$$q_{i,j} = \frac{D_{i,j}}{LT}, \tag{1}$$

where $D_{i,j}$ is the total distance traveled by all vehicles within the network during analysis period i on day j (veh-km), L is the total length of the network (lane-km) and T is the length of the analysis period (h). The density of the vehicles within the network is also given by:

$$k_{i,j} = \frac{T_{i,j}}{LT}, \tag{2}$$

where $T_{i,j}$ is the total time vehicles spend traveling within the network during analysis period i on day j (veh-h).

Define D_i as the average distance traveled within the network during a given analysis period i over all J days. The actual distance traveled during that analysis period on any particular day j, $D_{i,j}$ can be written as a function of this mean value:

$$D_{i,j} = D_i + \epsilon_{i,j}, \tag{3}$$

where $\varepsilon_{i,j}$ has mean 0 and variance $\sigma^2_{\epsilon_i}$ for a particular period i. In this construction, $\varepsilon_{i,j}$ represents how much extra or fewer veh-km are traveled during interval i on day j compared with the mean over all J days; i.e., the daily fluctuations in distance traveled during that analysis period. We expect that the day-to-day variations in total distance traveled or total time spent traveling are not a function of the day itself and cannot be predicted in advance, but instead occur as differences in travel behavior exhibited in drivers over different days. As will be shown, these assumptions help to facilitate the derivation of a simple formula to determine the day-to-day variance in travel time.

Similarly, define T_i as the average time vehicles spend traveling within the network during analysis period i across all J days. The actual time spent traveling on any particular day j can be written as the sum of the mean value and a term representing day-to-day fluctuations, $\gamma_{i,j}$:

$$T_{i,j} = T_i + \gamma_{i,j}, \tag{4}$$

where $\gamma_{i,j}$ has mean 0 and variance $\sigma^2_{\gamma_i}$ for a particular period i.

From Eq. (1), the expected average flow on the network for a particular analysis period i measured across all J days is:

$$q_i = \mathrm{E}_j(q_{i,j}) = \mathrm{E}_j\left(\frac{D_i + \epsilon_{i,j}}{LT}\right) = \frac{D_i}{LT}, \tag{5}$$

where E_j represents the expectation taken over the variable j. The day-to-day variance in average flow during this period is:

$$\sigma^2_{q_i} = \mathrm{Var}_j(q_{i,j}) = \mathrm{Var}_j\left(\frac{D_i + \epsilon_{i,j}}{LT}\right) = \frac{\sigma^2_{\epsilon_i}}{(LT)^2}, \tag{6}$$

where Var_j denotes the variance taken across the variable j.

Similarly, the expected average density on the network for a particular analysis period i over J days is:

$$k_i = \mathrm{E}_j(k_{i,j}) = \mathrm{E}_j\left(\frac{T_i + \gamma_{i,j}}{LT}\right) = \frac{T_i}{LT}, \tag{7}$$

and the day-to-day variance in average density during this period is:

$$\sigma^2_{k_i} = \mathrm{Var}_j(k_{i,j}) = \mathrm{Var}_j\left(\frac{T_i + \gamma_{i,j}}{LT}\right) = \frac{\sigma^2_{\gamma_i}}{(LT)^2}. \tag{8}$$

As per Edie's definitions, the generalized (space-mean) velocity that relates flow to density during analysis period i on day j, $v_{i,j}$, is the ratio of $q_{i,j}$ to $k_{i,j}$:

$$v_{i,j} = \frac{q_{i,j}}{k_{i,j}} = \frac{D_{i,j}}{T_{i,j}}. \tag{9}$$

Of more interest here, however, is the inverse of generalized velocity—generalized pace. The generalized pace during time period i on day j, $p_{i,j}$, is also equivalent to the average travel time per kilometer of all vehicles in the network during that period (i.e., the instantaneous mean travel time per distance traveled during that period). The generalized pace defined in this way is also equivalent to the weighted arithmetic mean of individual per kilometer travel times if the time each vehicle spends traveling in the network is used as the weights. With this definition, the average per km travel time is the inverse of the space-mean speed measured by Eq. (9). For the remainder of this paper, travel time on a network will refer to the normalized travel time per kilometer measured in this way[1]. This travel time can be expressed as:

$$p_{i,j} = \frac{k_{i,j}}{q_{i,j}} = \frac{T_{i,j}}{D_{i,j}}. \tag{10}$$

Equations (3) and (4) can be substituted into Eq. (10) to describe the average travel time as a function of daily fluctuations in the total time and distance vehicles traveled within the network. This yields:

$$p_{i,j} = \frac{T_i + \gamma_{i,j}}{D_i + \epsilon_{i,j}} = \frac{T_i}{D_i}\left[\frac{1 + \frac{\gamma_{i,j}}{T_i}}{1 + \frac{\epsilon_{i,j}}{D_i}}\right] \approx \frac{T_i}{D_i}\left(1 + \frac{\gamma_{i,j}}{T_i}\right)\left(1 - \frac{\epsilon_{i,j}}{D_i}\right), \tag{11}$$

where the approximation holds by using a first-order Taylor Series expansion around zero to describe the ratio in brackets. Equation (11) can be further approximated by:

$$p_{i,j} \approx \frac{T_i}{D_i}\left(1 + \frac{\gamma_{i,j}}{T_i} - \frac{\epsilon_{i,j}}{D_i}\right), \tag{12}$$

since the $\gamma_{i,j}\,\varepsilon_{i,j}\;/\,(T_i\,D_i)$ term should approach zero if the daily fluctuations in total time and distance traveled by vehicles within the network during the analysis period i are small compared to the mean values for that period.

From Eq. (12), the average travel time, p_i, and daily variation in travel time during period i, $\sigma^2_{p_i}$ can be expressed as:

$$p_i = \mathrm{E}_j(p_{i,j}) = \frac{T_i}{D_i}, \tag{13a}$$

$$\sigma^2_{p_i} = \mathrm{Var}_j(p_{i,j}) = \frac{1}{D_i^2}\left(\sigma^2_{\gamma_i} + \sigma^2_{\epsilon_i} p_i^2 - 2p_i \times \mathrm{cov}(\epsilon_i, \gamma_i)\right) \tag{13b}$$

Notice that Eqs. (13a) and (13b) are functions of T_i and D_i, and in turn average flow and average density during any particular period i. It follows then that the patterns between p_i and $\sigma^2_{p_i}$ would be related to patterns between q_i and k_i.

Relationship between day-to-day travel times and the MFD

We now examine the expected relationship between mean and day-to-day variance of travel time across a typical urban network. We focus our discussion on cases in which the aggregate flow–density exhibits a clockwise hysteresis pattern, since this type of pattern has been shown to be very likely when drivers do not have perfect real-time information to adaptively re-route to avoid localized congestion Gayah and Daganzo (2011).

[1] Note that we consider here only the instantaneous travel time as defined in this manner. Actual travel times of individual vehicles whose trips span multiple analysis periods (i.e., individual vehicles "experienced" travel times) can be determined by taking a weighted average of the instantaneous travel times using the amount of time spent in each period as a weight.

Consider now a reproducible clockwise hysteresis pattern in the relationship between flow and density of a network which may be observed during a typical rush period on any day j; see Fig. 3. If the same general pattern is repeated over all J days, the flow–density relationship averaged across all J days would also have a similar shape (we refer to this as an average MFD). Also shown on Fig. 3 is a line originating from the origin with a constant slope that indicates some average travel time per kilometer, p. Consider the two points on the curve associated with this mean travel time p; i.e., the points where this line intersects the flow–density curve. The point on the top right is associated with increasing density (i.e., the beginning of the rush period or onset of congestion) while the point on the bottom left is associated with decreasing density (i.e., the end of the rush period or dissipation of congestion). For simplicity, we refer to these periods as $i = b$ and $i = e$, respectively. Clearly, $k_b > k_e$ and $q_b > q_e$ for any valid value of p if the MFD exhibits a clockwise hysteresis loop.

For now, assume that the day-to-day variations in $D_{i,j}$ and $T_{i,j}$ follow the same distribution for both analysis period $i = b$ and $i = e$; i.e., $\sigma_{\epsilon_b} = \sigma_{\epsilon_e} = \sigma_e$, $\sigma_{\gamma_b} = \sigma_{\gamma_e} = \sigma_\gamma$ and $\text{cov}(\varepsilon_b, \gamma_b) = \text{cov}(\varepsilon_e, \gamma_e) = \text{cov}(\varepsilon, \gamma)$. The variance in travel time for each of these time periods can be calculated using Eq. (13b). We now examine the difference in the travel time variance between these two time periods to see if any systematic differences emerge between the beginning and end of the rush. This difference can be written mathematically as:

$$\sigma^2_{p_b} - \sigma^2_{p_e} = \left[\frac{1}{D_b^2} - \frac{1}{D_e^2}\right]\left(\sigma^2_\gamma + \sigma^2_\epsilon p^2 - 2p \times \text{cov}(\epsilon, \gamma)\right). \quad (14)$$

The term in parentheses will always be greater than zero by definition; otherwise, this would imply that the travel time variance during either of the two time periods would be less than zero, which is not possible. Therefore, the sign of $\sigma^2_{p_b} - \sigma^2_{p_e}$ would follow the sign of the term in brackets. Clearly, this term will be positive if

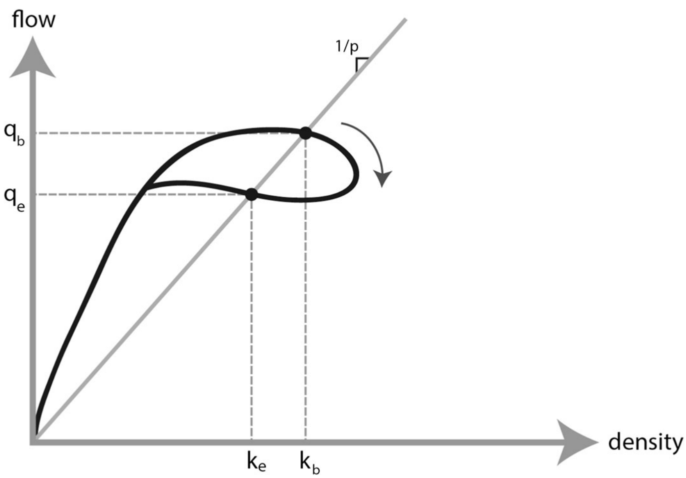

Fig. 3 Clockwise hysteresis loop on the MFD

$D_b < D_e$ and will be negative if $D_b > D_e$. Examining Eq. (1) and Fig. 3, we find that when the MFD exhibits a clockwise pattern then $D_b > D_e$. Therefore, $\sigma^2_{p_b} - \sigma^2_{p_e} > 0$ when reproducible clockwise hysteresis loops exist in the MFD, and variance of day-to-day travel times would be greater as the network recovers from congestion than during the onset of congestion. This results in a relationship between the mean and variance of travel time which follows an anti-clockwise hysteresis pattern. Thus, we see that daily clockwise hysteresis loops in the MFD are connected to anti-clockwise hysteresis loop patterns between the mean and day-to-day travel time variance.

The preceding analysis finds that anti-clockwise hysteresis loops would exist in the relationship between mean and variance of daily travel times if the distribution of daily fluctuations in time and distance traveled is the same during both total time periods $i = b$ and $i = e$. However, this assumption is restrictive and should not be expected to be true in general. In fact, the theoretical work in Daganzo et al. (2011) and Gayah and Daganzo (2011) suggest that variance in observed flows and densities would be greater during the dissipation of congestion than during the onset of congestion (i.e., $\sigma^2_{q_b} < \sigma^2_{q_e}$ and $\sigma^2_{k_b} < \sigma^2_{k_e}$). The reason for this is the instability that arises in congested multi-route networks that causes vehicles to distribute themselves more unevenly across the network with time. These uneven distributions are unpredictable and result in less predictable flows and densities as time passes. Thus, over the course of many days one would expect more reproducible network behavior during the beginning of the rush than during the end of the rush. Based on Eqs. (6) and (8), we would expect that $\sigma^2_{\epsilon_b} < \sigma^2_{\epsilon_e}$ and $\sigma^2_{\gamma_b} < \sigma^2_{\gamma_e}$. Examining Eq. (13b), we find that in this more realistic case the difference in travel time variance between $i = b$ and $i = e$ would only be exacerbated. Therefore, we would expect the relationship between mean and variance in travel time to follow an even stronger anti-clockwise hysteresis loop.

It is also important to note that in more realistic cases $\mathrm{cov}(\varepsilon_{b,j}, \gamma_{b,j}) \neq \mathrm{cov}(\varepsilon_{e,j}, \gamma_{e,j})$. If this covariance term is larger during the end of the rush than during the beginning of the rush, then this could diminish the anti-clockwise loops previously predicted. However, there is no a priori indication that this is the case. Even if $\mathrm{cov}(\varepsilon_{b,j}, \gamma_{b,j}) < \mathrm{cov}(\varepsilon_{e,j}, \gamma_{e,j})$, the difference between $\mathrm{cov}(\varepsilon_{b,j}, \gamma_{b,j})$ and $\mathrm{cov}(\varepsilon_{e,j}, \gamma_{e,j})$ would have to be very large to eliminate the anti-clockwise pattern. Therefore, we would still expect that this pattern should occur in general.

Verification using simulation

The previous theoretical analysis shows that anti-clockwise loops should be expected in the relationship between the mean and variance of travel times during a typical rush period. However, the derivations relied on some key simplifying assumptions. For example, we use a first order Taylor Series expansion to model the travel time variance and model daily fluctuations in distance and time traveled as random 'error' terms away from the mean values. In this section, we apply Eq. (13b) to realistic trajectory data from a micro-simulation to verify its validity and confirm the hysteresis pattern predicted from theory.

Study data

Trajectory data are obtained from a simulation of downtown Orlando, Florida, coded in the VISSIM micro-simulation software. The network was created for use as a part of a study for the City of Orlando (Dixit et al. 2009). The simulated network contains a portion of the Orlando street network, shown in Fig. 4; it covers a roughly 2.7 × 2.7 km area and contains 107 signalized intersections. Note that about 40 of these signals lie on the perimeter of the network, which implies that signal spacings in the interior of the network are about 0.34 km. Also note that there are about an equal number of unsignalized intersections within the network that occur at minor street crossings. Overall, speed limits (and thus average travel speeds) are generally low.

The network was created using traffic data obtained from the City of Orlando. In addition, the network was calibrated to match empirical data within the city obtained by mobile probe vehicles, or "chase cars". These chase cars followed individual vehicles randomly as they traveled within the network boundaries and recorded data on the time spent moving, time spent stopped and total travel distance. Parameters of the macroscopic two-fluid model (Herman and Prigogine 1979; Ardekani and Herman 1987) were calculated for the chase cars and then these parameters were replicated within the simulation. In this way, the macroscopic traffic properties of the real downtown Orlando network were accurately reflected within the simulation package. Full details on the calibration and validation procedure can be found in Dixit et al. (2009, 2011).

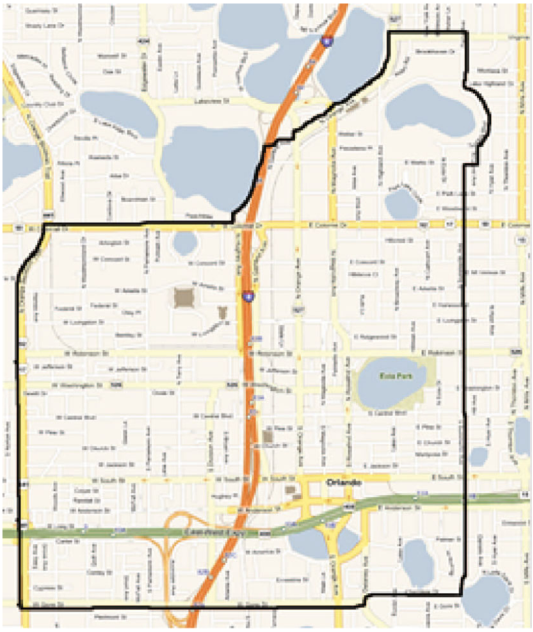

Fig. 4 Orlando downtown network (ref: http://local.live.com/, 20 April 2007)

The simulated period was 3 h long and consisted of a ½-h "warm-up" period, two 1-h rush periods, a ½-h "cool-down" period. The simulation was run 28 times to emulate the day-to-day variations that might be experienced across days in which travel demands and patterns were the same (e.g., consecutive Monday morning rush periods). Generalized flows, densities and travel times were calculated at discrete 120-s analysis periods. Different analysis period lengths were also tested to see if they significantly changed the observed behavior; however, the general trends and results were found to be insensitive to the analysis period length.

Verification of hysteresis

To verify the behavior expected from the theoretical formulations provided in the preceding sections, we first verify that the aggregate flow–density relationship over many simulation runs (each representing a different 'day') exhibits a clockwise pattern. Figure 5 presents the aggregate relationship averaged over the 28 simulation runs (i.e., the average MFD) for the first rush hour period (between 1,800 and 5,400 s of simulation time). Notice the very clear clockwise pattern between flow and density, where flows are consistently higher as density increases than as density decreases. Individual days follow a similar pattern with some stochastic variation. Also notice that this pattern is observed even though the network does not become incredibly congested during the rush (i.e., does not enter the down-sloping branch of the MFD), although the network does reach its maximum capacity of about 375 vehicles/h-lane.

The simulation data can also be used to calculate the variances, $\sigma^2_{\epsilon_i}$, $\sigma^2_{\gamma_i}$ and cov $(\varepsilon_i, \gamma_i)$ during the rush period. Figure 6 presents the relationship between the mean and variance of density and flow, respectively. As per Eqs. (6) and (8), $\sigma^2_{\epsilon_i}$ and $\sigma^2_{\gamma_i}$ are proportional to the variance in density and flow, respectively. Notice in Fig. 6 that both of these plots show a clear anti-clockwise hysteresis trend where the variance in density and variance of flow are both higher during the end of the rush than

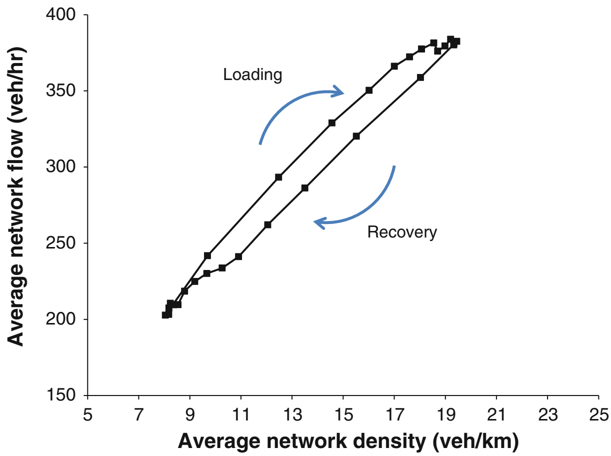

Fig. 5 Aggregate flow–density relationship during the first rush hour period of the simulation

during the beginning of the rush. This is as expected from the theory presented in Gayah and Daganzo (2011), and has been empirically observed in Geroliminis and Sun (2011). The cov(ε_i, γ_i) is also determined from the trajectory data for each analysis period. The individual covariance values ranged from 12 to 58 km-h, which were associated with correlation coefficients of 0.28–0.94. This shows that strong positive correlation exists between these two terms, as expected.

Based on the clockwise loop pattern in the aggregate flow–density relationship shown in Fig. 5 and the higher daily fluctuations in time and distance traveled implied by Fig. 6, we would expect a clear anti-clockwise hysteresis pattern in the relationship between mean and variance of travel time. To confirm this, we first determine the theoretical relationship between mean and variance in travel time that would be expected based on Eq. (13b). This relationship is plotted as a dotted line in

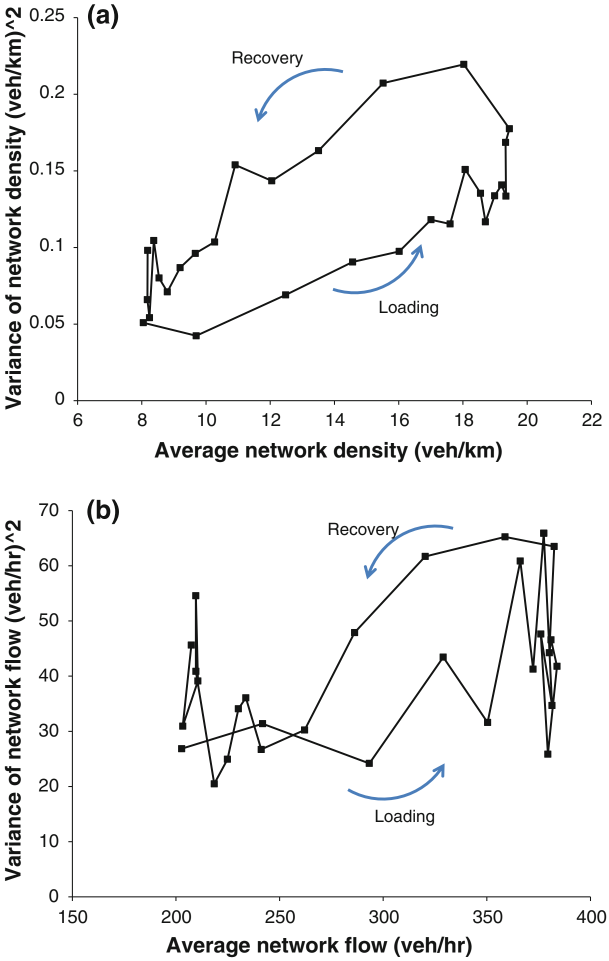

Fig. 6 Relationship between mean and variance of: **a** network density; and **b** network flow, during first rush hour period

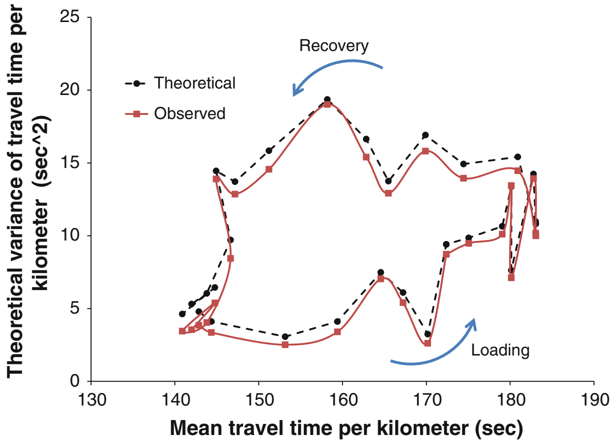

Fig. 7 Predicted and observed relationship between mean and variance of travel time for the first rush hour period

Fig. 7. Notice from Fig. 7 that the predicted relationship follows the pattern expected from the theory in "Relationship between day-to-day travel times and the MFD"—travel time variance is consistently higher as the network recovers from congestion than during the onset of congestion.

The predicted relationship between mean and variance of travel time is then compared to the actual relationship calculated directly from the 28 'days' of simulation data; this latter relationship is also plotted as a solid line in Fig. 7. Notice that the observed data confirms the anti-clockwise pattern that was expected from the theoretical analysis. In addition, notice that the predicted line follows the observed line almost exactly. This verifies the validity of the formulation and modeling assumptions in "Theoretical model" as Eq. (13b) seems to predict the observed travel time variance rather well. In general, the predicted values of travel time are within 14 % of the observed values and the maximum difference is about 2 s^2. The small differences between Eq. (13b) and the observed simulation data could be attributed to the use of a first-order Taylor Series expansion (instead of a higher order approximation) in Eq. (10) and the removal of the $\gamma_{i,j}\ \varepsilon_{i,j} / (T_i\ D_i)$ term from Eq. (11). However, even with these approximations, the analytical equation fits the data very well and can be used to estimate daily travel time variation using just measured flows and densities in a network across many days.

The same methodology was applied to the second rush hour that occurred during the simulation period (between 5,400 and 9,000 s of simulation time). The findings were consistent with the previous results; see Fig. 8.

Concluding remarks

This study examines the dynamics of urban traffic networks to help understand the relationship between mean and variation of day-to-day travel times. A simple

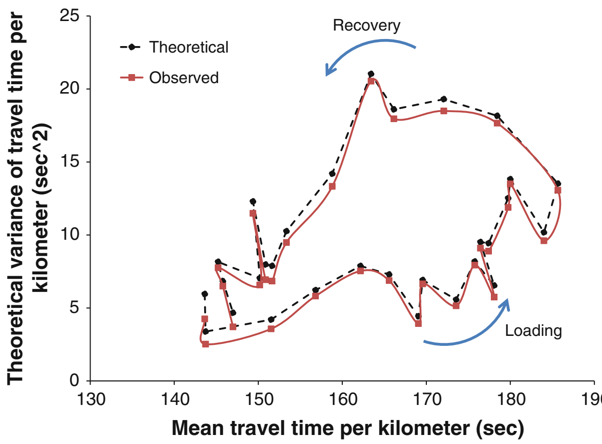

Fig. 8 Predicted and observed relationship between mean and variance of travel time for the second rush hour period

theoretical framework is presented to analytically determine the variance of day-to-day travel times as a function of average flow and average density, as well as daily variation in total distance traveled and total time spent traveling within the network. Note that the latter two terms can be estimated through the variation in flow and density during the same period over many days. Using this formulation, we show that travel time variance is expected to be higher as congestion dissipates in a network (i.e., as mean travel times decrease) than during the onset of congestion (i.e., mean travel times increase) if a clockwise pattern exists in the MFD of the network. This anti-clockwise hysteresis pattern in the relationship between mean and variance of travel time would be expected both in the simplistic case when daily errors in time and distance traveled have the same distribution, and the more likely case when the errors are larger during the end of the rush than during the beginning of the rush. Clockwise hysteresis patterns in the MFD are to be expected even under the most idealistic case of symmetric loading on homogeneous networks because drivers do not have perfect information to adaptively re-route to avoid congestion Gayah and Daganzo (2011). This implies that the anti-clockwise hysteresis between mean and variance of travel time should not be uncommon in urban networks. Travel demand and behavior models that include travel time reliability need to recognize the fact that travel time variance changes in this way throughout the rush period to accurately describe the effects of reliability on driver behavior.

Recent field data does indeed exhibit this predicted pattern (Bates et al. 2003). Additionally, we have verified the existence of the predicted pattern using a micro-simulation of the urban street network of Orlando, Florida. The simulated network contains a clockwise relationship between average flow and density (as expected from the model). The resulting relationship between mean and variance of travel time from empirical data matches very closely that predicted with the theoretical framework. This result helps justify the assumptions made in the theoretical analysis. Note that the simulated conditions assume demands and travel patterns are rather similar, representing multiple unique periods (i.e., consecutive Monday mornings). In the more likely case that we are concerned with travel time variances across different consecutive days of the week when demands and travel patterns would vary, the anti-clockwise pattern should be even more prevalent. This results from the fact that daily fluctuations in total time and distance traveled in the network during any given analysis period would only increase.

While this study does predict and confirm the anti-clockwise hysteresis pattern in the relationship between mean and variance of travel time in the case of a clockwise hysteresis loop in the MFD, it does not provide much insight into the relationship expected when the MFD has a different pattern. Gayah and Daganzo (2011) shows that four MFD patterns are possible. While two of them are highly unlikely (anti-clockwise loop or figure-8 pattern), the single path pattern show be expected if drivers have sufficient information and ability to adaptively re-route to avoid locally congested areas. Given the theoretical framework in "Theoretical model" we would expect that for a single path pattern where $k_b = k_e$ and $q_b = q_e$, hysteresis would not exist between mean and variance of travel time. However, this prediction has not been verified in this paper. Further work seeks to examine the relationship between mean and daily variance of travel time in this scenario. Future work should also be performed to quantify the magnitude of the hysteresis loops in travel time variance and its relationship to the magnitude of the hysteresis loops in the MFD. Additionally, future research will aim to examine how variations in instantaneous travel time impact variations in experienced travel time, as well as the variation of travel times between individual vehicles on a given day. A complete understanding of all of these types of travel time variances can lead to improvements in current travel demand models, which will help increase our understanding of traveler behavior in urban networks.

Acknowledgments We would like to thank three anonymous reviewers for their comments and suggestions that helped to improve this paper

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