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Behavioural foundations of two-fluid model for urban traffic

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ABSTRACT

Traditionally traffic flow models have been based on analogous physical phenomena. Though these models have been successful in representing traffic flow, there is a need to provide a systematic behavioural explanation for their existence. One such model is the two-fluid model which is analogous to the principles of Bose–Einstein condensation of particles at low temperatures. The model has been extensively used to characterize the quality of traffic on urban networks and arterial streets. The two parameters of the model essentially represent 'free flow' travel time and level of interaction among vehicles. Though the studies have found the parameters of the two-fluid model to be significantly correlated with driver behaviour (aggressive/conservative) and crash rates, no systematic behavioural explanation has been found. This paper proposes a behavioural framework based on individual trade-off behaviour to explain the two-fluid model phenomenon. The two-fluid model is derived based on a driver's attempt to maximize his quality of travel, by travelling fast while maintaining safety. Contrary to earlier assumptions the proposed framework shows the two parameters to be correlated. The theoretical framework was tested using two-fluid model data from various cities. The data was also used to estimate the effects of geometric factors on the perception of likelihood of a crash and the severity of the crash that affect the two-fluid model. Increase in the fraction of one-way streets was found to reduce the driver's perception of likelihood to crash. While reduction in the fraction of one way streets and increase in average number of lanes per street, signal density and fraction of actuated signals increased the perceived level of severity of a crash.

1. Introduction

The two-fluid model (Prigogine and Herman, 1971) was developed from kinetic theory of vehicular traffic, and is based on the principles of Bose–Einstein condensation of particles at low temperatures. This model has been widely used to characterize traffic flow on urban networks (Ardekani, 1984; Vo et al., 2007).

Though field studies (Herman et al., 1988) have shown that individual driver behaviour (conservative/aggressive) has a significant effect on the parameters of the two-fluid model, so far there has been no theory providing a behavioural explanation.

The development of the two-fluid model was based on an analogous physical phenomenon of splitting of molecular distribution functions into the ground state and excited state (Prigogine and Herman, 1971). This phenomenon is also referred to as Bose–Einstein condensation in physics. Similar analogies (for example fluid dynamics) have provided insights into the complex working of traffic, and have been used to develop useful engineering solutions. The possible set of such solutions are dependent upon the construct of the theory. For instance, it would not be possible to develop traffic engineering solutions

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targeted towards behaviour, under a theory which is *purely* based on fluid flow or particles. Though traffic flow models have been found to be affected by driver behaviour, they are predominantly based on fluid dynamics or particle physics and have minimal consideration for driver's perception and attitudes. Therefore most traffic engineering solutions that use these traffic models do not have the ability to consider impact on driver behaviour or safety. Traffic flow models that are based on driver behaviour will be useful to capture effect of road conditions, various road infrastructure, education and insurance strategies on drivers risk attitudes and perception. This understanding can then be used to develop policy and engineering solutions for promoting safe driving.

The macroscopic fundamental diagram has also been shown to be affected by behaviour. Daganzo (2002a, 2002b) proposed a behavioural model based on rabbits (aggressive drivers) and slugs (conservative drivers) for multilane traffic flow to explain the lambda shape of the fundamental diagram, propagation of disturbances and certain characteristics of traffic flow at a merge. This was further verified by field data by Banks et al. (2003). Hamdar et al. (2008) proposed a car following model based on prospect theory and risk aversion, and identified the effects of different risk attitudes on the Macroscopic Fundamental Diagram. Recently, Chen et al. (2012a, 2012b) identified aggressive and timid behaviour in car following behaviour in NGSIM data and strong correlation in these behaviours before and during traffic oscillations. They also found that there is a need to properly model these behaviours in microscopic simulation models. This provides further motivation to develop behavioural foundations for other traffic flow models that have had their origins in the physics of traffic.

This paper presents a derivation of the two-fluid model based on assumptions regarding the utilities of the driver that are motivated by findings in literature and tests the validity of these assumptions by utilizing historical data. The following section describes the two-fluid model.

2. Background

2.1. The two-fluid model

The two-fluid model assumes that vehicular traffic flow in an urban network or street can be understood as consisting of stopped and running vehicles. The model describes the relationship between the vehicles' running speed (v_r) and the fraction of running vehicles (f_r).

$$v_r = v_m (f_r)^n \tag{1}$$

where v_r is the running speed of the vehicles; f_r is the fraction of running vehicles; v_m *is* the maximum speed (corresponding to minimum travel time); n represents one of the two-fluid model parameters.

In general, the subscript '*r*' indicates that the variable is measured for vehicles that are running and the subscript '*s*' indicates that the variable is measured for vehicles that are stopped, subscript '*m*' is used for the maximum speed or minimum travel time per mile.

When all vehicles are running (i.e. there are no stopped vehicles in the network, $f_r = 1$), therefore the running speed (v_r) is equal to v_m. Therefore v_m is defined as the average maximum speed, when no vehicles are stopped.

$$T_m = \frac{1}{v_m} \quad (T_m \text{ is then the minimum free flow travel time per mile})$$

Ardekani (1984), through field experiments, showed that it was possible to characterize urban networks by the two-fluid model. The study also used aerial photographs to validate the ergodic assumption of the two-fluid model. The ergodic assumption states that the ratio of running time per mile (T_r) to the travel time per mile (T) is equal to the ratio of the number of vehicles running to the total number of vehicles (Eq. (2)). Where, T_r and T are the inverse of v_r and v respectively.

$$f_r = \left(\frac{T_r}{T}\right) \tag{2}$$

Utilizing Eqs. (1) and (2) and that the running speed is equal to the inverse of the running time per mile.

$$T_r = T_m^{\frac{1}{n+1}} T^{\frac{n}{n+1}} \tag{3a}$$

$$\ln T_r = \frac{n}{n+1} \ln T + \frac{1}{n+1} \ln T_m \tag{3b}$$

Eq. (3) describes the relationship between the average travel time per mile (T) and the running time per mile (T_r). n and T_m are parameters of the two-fluid model and determine the sensitivity of the travel time per mile to the stopped time per mile. These parameters have been used in literature to describe the quality of traffic on the networks (Ardekani, 1984). The travel time per mile is defined as the sum of the running time per mile and the stopped time per mile (T_S). Hence Eq. (3) can be rewritten as:

$$T_S = T - T_m^{\frac{1}{n+1}} T^{\frac{n}{n+1}} \tag{4}$$

When the stopped time per mile diminishes to zero, the travel time per mile is equal to the running time per mile ($T = T_r$). Replacing $T = T_r$ in Eq. (3) results in the travel time per mile being equal to T_m. Therefore, T_m could be considered as the free-flow travel time per mile when there are no stops. A high value of T_m is indicative of low free flow speeds. While a high value of n implies that a unit increase in stopped time per mile (increase in congestion) would result in a larger increase in travel time. There were a series of simulation studies (Mahmassani et al., 1984, 1987, 1990) that studied the application of the two-fluid model to networks.

2.2. Two-fluid model and driver behaviour

The parameters of two-fluid model have also been used to characterize driver behaviour. Herman et al. (1988) studied the effects of extreme driver behaviours on the two-fluid model. They found that a test car driver instructed to drive *aggressively* in a network established a significantly different individual two-fluid trend than one instructed to drive *conservatively* in the same network at the same time. The two-fluid model was also generated by following randomly selected vehicles in the network, which was considered to be 'normal' or representative of the overall network. Aggressive drivers were found to have a lower T_m than normal drivers and normal drivers had lower T_m than conservative drivers. They also found that all three two-fluid trends (aggressive, conservative, and normal) converged as the stopped time per mile increased, implying that n would be higher for aggressive drivers compared to normal drivers, and conservative drivers would have the lowest value of n. Therefore the level of aggressiveness of the driver and T_m are negatively correlated, and n and level of aggressiveness are positively correlated. This is shown in Fig. 1. Herman et al. (1988) found the trends for these three behaviours to be statistically different.

Recently, Dixit et al. (2011) found that the two parameters of the two-fluid model for arterial streets were significantly correlated with and rear-end crash rates as well as severe crash rate. Additionally, they found that the two parameters were correlated between themselves as well. These suggest the two-fluid model captures certain behavioural characteristics that do have an impact on crash rates and crash severity, and this motivates the development of a behavioural framework.

Ayadh (1986) and Ardekani et al. (1992) selected various network environmental factors (Signal density, Average number of lanes, fraction of one way streets, cycle lengths of signals, etc.) and estimated regression models to understand the effects of these network features on T_m and n. The finding that network features (Ayadh (1986), Ardekani et al. (1992)) and driver behaviour (Herman et al. (1988)) affect the characteristics of the two-fluid model, suggests that the two-fluid model is able to capture the interrelationship between network characteristics and behaviour. The differences in network characteristics affect behaviour, which are in turn observed in the two-fluid model. The flow of information that affects behaviour is channelled through the process of attention, perception decision-making and prediction (Rumar, 1985; Wickens and

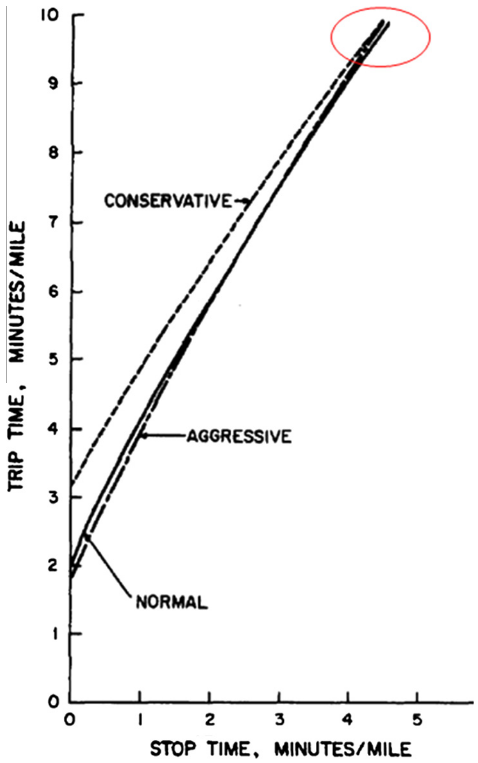

Fig. 1. The two fluid trends for aggressive, normal and conservative drivers in the Austin CBD (Herman et al., 1988).

Hollands, 2000; Fuller, 2005). An individual driver‘s decision is based on the perception of the physical realities. Since the driver’s behaviour is based on the perception of the physical attributes of the network, it would not be possible to separately identify whether the drivers’ behaviour a consequence of difference in perception or difference in network features. This essentially translates to a problem of two variables (perception and network features) and one observable variable (driver behaviour), it would not be possible to separately identify which of the two variables caused the variation in the observed variable. It would be best to study the two variables in conjunction. Therefore, the two-fluid model is being modelled as a psychophysical phenomenon.

3. Behavioural framework

Savage (1954) introduced a standard analytical framework to model individual decision making under uncertainty. The framework consists of three primitives’ states, consequences and acts. Based on Arrow’s (1981) definition states can be interpreted as mapping of a set of acts to consequences. In addition, by definition the states of the world are mutually exclusive and jointly exhaustive.

The state space associated to driving can be defined based on S = {crash, no crash}. In driving, the states of crash and no crash adhere to being mutually exclusive and jointly exhaustive, and maps the act of choosing a speed to the consequences associated to these two states. In the case of ‘crash’ the consequence is defined by the level of impact and in the ‘no crash’ it is the speed of getting to the destination. The driver attempts to choose a speed such that they reach the destination as *fast* and *safely* as possible.

An individual decision maker’s preferences are state dependent, when the consequences of the states of nature are of concern to the decision maker. For instance, in the case of driving, the two states of nature of direct consequence to the driver is crash or no crash; the consequence of a crash is a function of impact velocity, while of no crash is the driving at a certain average speed.

The state dependent utility approach has been widely used for insurance purposes (Parkin and Wu, 1972; Dreze, 1987), where it is believed that consumption of a certain commodity is dependent on the mutually exclusive states. A detailed note on State Dependent Preferences is provided by Karni (2005).

Recent work by Tarko (2009) used a utility-based framework to model a driver’s preferred speed as tradeoff between time gain and safety. The study assumed a (dis)utility over value of time, loss by driving slowly, perceived risk (of a crash) and perceived enforcement (risk of getting a traffic ticket).

This paper, utilizes a ‘‘state-dependent’’ utility approach under uncertainty to understand a traffic phenomenon that has been understood thus far only as an analogous physical ‘two-fluid’ system.

Perceived likelihood of these two states play a crucial role in determining how a driver behaves. Perception is the representation of physical reality in drivers’ minds. Since perception is internalized in drivers’ brains, it is extremely difficult to observe perception, i.e., the perceived likelihood of these two states directly. The observable quantities are the physical inputs (in the form of driving environment, traffic signal, etc.) to the driver and their reactions. Explanation of observable reactions (i.e., outputs) based on structured assumptions regarding perception (as functional transformations of physical inputs) provides us a better understanding of how people perceive and react. In this study functional forms are assumed for the perceived probability to crash, utility of not crashing and the utility of crashing.

The Transportation Research Board (1998) states that:

> ‘‘*Drivers’ speed choices impose risks that affect the probability and severity of crash.*’’ Elvik et al. (2004) on the basis of this and in conjunction with similar findings from Europe (European Commission, 1999) concluded that speed had an impact on the risk associated with

(1) probability of crashes
(2) severity of crashes

Fuller (2005) in his work to develop a general theory of driver behaviour states that:

> ‘‘*Once a motor vehicle begins to move, collision is not a refined estimate of a very low probability: it is inevitable. The driver more-or-less continuously makes direction and speed adjustment to avoid this otherwise certain outcome.*’’

In the case of urban networks, avoidance of collision is stopping before colliding with the vehicle or other roadside objects. Therefore, it can be posited that when the driver is stationary the driver has no reason to believe that they would cause a crash, Hence, under the two-fluid model framework, the perceived probability of causing a crash would be equal to zero when the fraction of time spent running is zero. The driver’s perceived probability of causing a crash (P_{crash}) increases as the fraction of time spent running (f_r) increases. Therefore, it is assumed that the driver’s perceived probability of causing a crash monotonically increases from zero (when $f_r = 0$) to a value α (when $f_r = 1$). The parameter α ensures that different drivers have different baseline perceptions of causing a crash, α is greater than zero and less than 1. β Reflects how a driver perceives the likelihood of a crash for a given fraction of time spent running. In this study, β is referred to as the perceived crash likelihood factor. It is assumed that the driver’s perceived probability of causing a crash is of the following form:

$$P_{crash} = \alpha\left(\frac{T_r}{T}\right)^{\beta}$$

or

$$P_{crash} = \alpha\left(\frac{v}{v_r}\right)^{\beta}$$

It is important to note that as β increases, the perceived probability of crash reduces. Therefore the higher the value of β, the lower the probability the driver believes of causing a crash. It is important to recognize that network characteristics can affect β. Based on the fraction of running time the perceived probability of crash may be written as follows:

$$\begin{aligned} &P_{crash} = \alpha f_r^{\beta} \\ &0 \leqslant \alpha f_r^{\beta} \leqslant 1 \\ &\beta > 0 \\ &0 < \alpha \leqslant 1 \end{aligned} \tag{5}$$

The utilities associated to the driver being in the state of "crash" and "no crash", are also based on the drivers perception. When the driver is in the state "no crash", the individual driver's utility increases if they reach their destination sooner, this can be accomplished by increasing their average speed. Similar to the formulation used by Tarko (2009), it is assumed that the utility of not crashing increases with increase in average speed.

$$u(no\ crash) = v \tag{6}$$

The disutility of crashing is assumed to be the perceived severity of an impact while driving at a given speed. It is a negative and monotonically decreasing power function of the running speed. If the running speed is zero the perceived severity of impact should be zero. As the speed increases the kinetic energy increases and therefore the impact would be more severe. This disutility of crash is the risk of severity identified by Elvik et al. (2004). Tarko (2009) used a similar formulation for disutility of crash (Tarko (2009), referred to it as "*perceived risk*"). Hence, it is assumed that the perceived utility of a crash is a negative power function of the running speed. Therefore,

$$u(crash) = -w(v_r)^k \quad \text{where } w > 0 \tag{7}$$

Different values of k can be attributed to the individual driver's perception of the severity of crash or confidence in their skill to survive in a severe crash. Since, k is the perceived impact factor and is associated to the kinetic energy it takes a value greater than 1. Tarko (2009) in his formulation of disutility of crash also constrained the power to be greater than 1. In the rest of the paper k is referred to as the perceived impact factor and w is the crash weighting factor.

Network features, and socio demographic characteristics (for example, older drivers or not) play an important role in affecting the choice of speed. These effects are captured through determining factors that affect the parameters associated to drivers' belief of the likelihood (β) of causing a crash and the perceived impact (k). In this particular study, this was achieved through regression analysis.

While driving, the driver perceived probability of being associated to the state of "crash" is represented by Eq. (5), and the perceived probability to be in the state of "no crash" is represented by $1 - P_{crash}$. Under these perceived probabilities, the driver attempts to increase his average speed ($U_{no\ crash}$) while trying to reduce severity of the impact of a crash (U_{crash}). This can be represented as the state-dependent expected utility (EU) of driving:

$$\mathrm{EU} = (1 - P_{crash}) \times u(no\ crash) + P_{crash} \times u(crash) \tag{8}$$

Using Eqs. (5)–(8), the expected utility can be written as:

$$\mathrm{EU} = \left(1 - \alpha f_r^{\beta}\right) v - \alpha f_r^{\beta} w(v_r)^k \tag{9}$$

Using Eqs. (3) and (9), the expected utility while driving can be written in terms of average speed and the running speed of the individual driver:

$$\begin{aligned} &\mathrm{EU} = \left(1 - \alpha\left(\frac{v}{v_r}\right)^{\beta}\right) v - \alpha w\left(\frac{v}{v_r}\right)^{\beta} (v_r)^k \\ &\mathrm{EU} = \left(1 - \alpha(v_r)^{-\beta}(v)^{\beta}\right) v - \alpha w(v)^{\beta}(v_r)^{k-\beta} \end{aligned} \tag{10}$$

There have been several studies (Greenshields (1935), Greenberg (1959), Drake and May (1967), Del Castillo (1995a, 1995b) and Wang et al. (2011)) in the area of traffic flow theory that have demonstrated the fundamental nature of the relationship between speed and flow, and developed models to fit these relationships. Essentially, it has been observed that the average speed reduces with increase in traffic density. Therefore it can be assumed that the average travel time per mile is exogenously determined by the traffic density (level of congestion). Therefore based on the number of vehicles around the driver, the average travel time per mile is pre-determined for the driver.

An aggressive driver could have a lower perceived probability of causing a crash or a lower perceived impact from crashing, therefore increasing their running speed by constant manoeuvring to change lanes and hard braking to avoid hitting vehicles in front. On the contrary a conservative driver would have a higher perceived probability to cause a crash or a higher perceived impact from crashing, resulting in reduction in their running speed.

Therefore, given the driver's average speed (v) for a particular trip, the choice of running speed (v_r) of the driver for this trip would be the result of the expected utility maximization (Eq. (10)).

$$\frac{\partial EU}{\partial v_r} = \alpha\beta(v_r)^{-\beta-1}(v)^{\beta+1} - \alpha(k-\beta)w(v_r)^{k-\beta-1}(v)^{\beta} \tag{11}$$

To determine v_r that maximizes Eq. (10), Eq. (11) is equated to zero.

$$\Rightarrow \beta v = (k-\beta)w(v_r)^k \tag{12}$$

An important corollary of Eq. (12) is that $k > \beta$. The behavioural implication of this can be seen by studying the marginal rate of change w.r.t. running speed for perceived probability of being involved in a crash and perceived disutility of being in a crash:

$$\frac{\partial P_{crash}}{P_{crash}\partial v_r} = -\frac{\beta}{v_r}$$

$$\frac{\partial u(crash)}{u(crash)\partial v_r} = \frac{k}{v_r}$$

The condition $k > \beta$ implies that the absolute value of marginal rate of change for the perceived disutility is larger than the absolute value of marginal rate of change for the perceived probability to crash. This suggests that in an urban network, a driver's perceived disutility of crashing changes at a much faster rate than the perceived probability to crash for a unit change in the running speed.The perceived probability described in Eq. (5) can be demonstrated to adhere to the empirical findings by Elvik et al. (2004), on the relationship between crashes as a power function of speeds. The power function suggests that the number of accidents increases as the average speed increases. Therefore, e is greater than zero.

$$\frac{\text{Accidents after}}{\text{Accidents before}} = \left(\frac{\text{Speed After}}{\text{Speed Before}}\right)^e \tag{13}$$

Using v_r from Eq. (12) and substituting it in Eq. (5), the perceived probability of a crash becomes a function of average speed (v):

$$P_{crash} = \left(\frac{w(k-\beta)}{\beta}\right)^{\frac{\beta}{k}} v^{\frac{\beta(k-1)}{k}} \tag{14}$$

Consider now implementation of a strategy that does not affect β, k or w (as seen later, there was no statistical evidence that differences in cycle time affect β, k or w), such as improvement in signal cycle timing found to affect that increases the average speed after (v_{after}) compared to the average speed before (v_{before}) the implementation of the strategy.

$$\frac{P_{crash\ after}}{P_{crash\ before}} = \left(\frac{v_{after}}{v_{before}}\right)^{\frac{\beta(k-1)}{k}} \tag{15}$$

As the number of vehicles travelling on the road are the same after the implementation of changes to the cycle time, the expected number of crashes increases after the speed increases if k is greater than 1. This was already assumed for Eq. (7) in the disutility of crashing, and was also considered by Tarko (2009). This ensures that the perceived probabilities are aligned with the objective risks, i.e. they increase with increasing average speed.

Comparing Eq. (13) with Eq. (15), it can be concluded that the power $\beta\,(k-1)/k$ in Eq. (15) can be thought of as a driver's estimate of the objective risk specified by the power 'e' in Eq. (13).

To ensure that k is greater than one, k is replaced by $\frac{n+1}{n}$, where $n > 0$. This transforms Eq. (12) into the standard two-fluid model form shown in Eq. (4).

$$\left(\frac{1}{v_r}\right) = \left(\left(\frac{(n+1-\beta n)}{n\beta}\right)^n\right)^{\frac{1}{n+1}}\left(\frac{1}{v}\right)^{\frac{n}{n+1}} \tag{16}$$

$$T_r = \left(\left(\frac{(n+1-\beta n)w}{n\beta}\right)^n\right)^{\frac{1}{n+1}} T^{\frac{n}{n+1}} \tag{17}$$

In Eq. (17) it is observed that if $T = T_r$ then the vehicle is in free flow conditions, and $\left(\frac{1+n-n\beta)w}{n\beta}\right)^n$ would be the drivers free flow travel time per mile (T_m).

$$T_m = \left(\frac{(1+n-n\beta)w}{n\beta}\right)^n \quad (18)$$

$$T_r = T_m^{\frac{1}{n+1}} T^{\frac{n}{n+1}} \quad (19)$$

Eq. (19) is the standard two-fluid model that is used in literature. Based on Eq. (18), T_m is a function of n, and its derivative with respect to n is:

$$\frac{dT_m}{dn} = \left(\frac{(1+n-n\beta)w}{n\beta}\right)^{n-1}\left(-\frac{w}{n\beta}\right) \quad (20)$$

T_m is a decreasing function of n, and therefore it should be expected that both are negatively correlated. This was also found in the data analyzed by Dixit et al. (2011).

Based on the utility maximization it was found that T_m and n are inversely related, i.e. if n increases T_m decreases and vice versa. This implies that an aggressive driver who has a lower free-flow travel time per mile would have a higher n. As described in the background section T_m reflects the free-flow travel time per mile, while n reflects how a unit increase in stopped time per mile affects the travel time per mile, therefore, a higher value of n suggests that a unit increase in stopped time per mile would result in a larger increase in travel time per mile.

The inverse relationship between T_m and n can be explained by the mechanistic argument. In free-flow conditions an aggressive driver can be expected to have a lower T_m than a conservative driver. In a congested state the travel time per mile and the stopped time per mile are constrained by the traffic around the driver. Therefore, to ensure that the two-fluid model trend converges at these congested conditions, a unit increase in stopped time would need to have a larger effect on travel time per mile for an aggressive driver than for a conservative driver. This implies that an aggressive driver would have a larger value of n as compared to a conservative driver. This was observed by Herman et al. (1988) in the convergence of the trend in the aggressive and conservative driver in Fig. 1, circled in red.[1] The framework provides a method to evaluate conservative and aggressive behaviour. Further, Ardekani (1984) showed that the Macroscopic Fundamental Diagram (MFD) can be derived from the two-fluid model, and therefore the finding of this study further indicate that driver behaviour would have an impact on the MFD.

The following section utilizes data collected by Bhat et al. (1992) to validate the theoretical relationship derived in Eq. (20). The regression models estimated by Ayadh (1986), Bhat et al. (1992) and Ardekani et al. (1992) to study the effects of these network features on T_m and n did not account for the relationship shown in Eq. (20). Considering them as independent parameters could result in estimates that do not appropriately reflect the true nature of the effects of the network features on the two-fluid model. The following section also presents a method based on least square estimation to determine the effects of network features on the perceptions of the drivers, this was achieved by estimating the effects of network characteristics on the perceived crash likelihood factor (β), perceived impact factor (k) and the crash weighting factor (w).

4. Results

Table 1 presents two-fluid data and network characteristics collected by Bhat et al. (1992) for various cities Table 1 also contains data for k which was recalculated as $(n + 1)/n$. The two-fluid models were developed, based on data collected using a chase car methodology. In the chase-car method adopted in all these studies, the driver follows a randomly selected vehicle until it leaves the network, parks or the chase requires unsafe and/or illegal manoeuvres, after which the next vehicle is selected for following. While transitioning from one vehicle to the next the vehicle is driven normally with respect to surrounding traffic. The two-fluid model estimated based on the per mile travel times recorded by the chase-car driver through this methodology is expected to be representative of the behaviour of drivers of the corresponding network.

In Table 1 the variables X_1 to X_{10} represent the network features, and are elaborated in Bhat et al. (1992):

- X_1: Average block length (miles),
- X_2: Fraction of one way streets,
- X_3: Average number of lanes per street,
- X_4: Intersection density (Number of intersections/square mile),
- X_5: Signal Density (Number of signalized intersections/square miles),
- X_6: Average speed limit, calculated by weighting based on length of streets (miles/h),
- X_7: Average cycle length during PM peak (seconds),
- X_8: Fraction of curb miles with parking allowed (miles),
- X_9: Fraction of actuated signals,
- X_{10}: Fraction of approaches with signal progression.

The relationship between T_m and n in Eq. (18) can be rewritten as:

[1] For interpretation of color in Fig. 1, the reader is referred to the web version of this article.

Table 1
Two-fluid parameters for cities (Bhat et al., 1992).

Network	Year	T_m	n	k	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}
Arlington 1	1990	1.98	1.06	1.94	496	0	2.8	108	17	30.6	94	0.375	1	0.537
Arlington 2	1991	1.95	0.61	2.64	479	0	2.9	124	25	30	91	0.315	1	0.557
Arlington 3	1991	1.77	0.66	2.52	479	0	2.9	124	22	30.6	93	0.315	1	0.516
Fort Worth 1	1990	2.52	0.88	2.14	300	0.71	3	355	268	30	75	0.36	0.11	0.69
Fort Worth 2	1991	1.71	1.2	1.83	557	0.01	4	31	5	36.1	66.7	0.215	1	0.048
Dallas 1	1983	2.12	1.36	1.74	350	0.4	2.6	160	80	30	80	0.218	0.324	0.328
Dallas 2	1990	2.79	0.77	2.30	338	0.65	3.2	282	224	30	80	0.131	0	0.488
Dallas 2	1991	2.77	0.8	2.25	340	0.67	3.1	270	215	30	84	0.13	0	0.29
Austin	1983	1.95	1.58	1.63	430	0.52	3	209	82	30.3	64	0.834	0	0.32
Austin	1990	2.14	1.46	1.68	409	0.43	2.9	175	82	30.2	68	0.834	0	0.32
Lubbock	1983	2.03	0.97	2.03	380	0.3	3.1	187	45	31.1	81	0.602	0.01	0.27
Lubbock	1990	1.78	1.27	1.79	446	0.15	3.1	153	30	31.1	97.1	0.602	0.02	0.37
Houston	1983	2.7	0.8	2.25	324	0.76	4.3	309	213	30	80	0.366	0	0.77
Houston	1990	2.24	1.11	1.90	324	0.76	4.3	309	213	30	80	0.366	0	0.77
San Antonio	1983	1.99	1.33	1.75	365	0.45	2.7	244	144	30	100	0.28	0.008	0.325
San Antonio	1990	2.52	1.14	1.88	365	0.41	2.6	240	150	29.8	89.3	0.28	0.008	0.395
San Antonio	1991	2.4	1.05	1.95	365	0.4	2.8	252	147	30	61.8	0.269	0.008	0.451
Albuquerque	1983	1.93	1.62	1.62	381	0.47	2.7	222	111	25.5	61	0.441	0.171	0.52
Albuquerque	1990	2.32	0.94	2.06	381	0.46	2.7	221	113	25.5	62.2	0.428	0.169	0.53
Mexico City	1983	1.72	1.63	1.61	359	0.72	3.9	225	47	31.2	120.0	0.110	0.000	0.613
Matamoros	1983	2.98	2.1	1.48	380	0.67	1.2	269	22	12.9	70	0.85	0	0

$$\Rightarrow (T_m)^{\frac{1}{n}} = \frac{w}{n\beta} + w\left(\frac{1}{\beta} - 1\right) \tag{21}$$

It should be noted that the two-fluid model parameters were collected at a network level, and the derivations presented in this paper were based on an individual driver. In this paper it is assumed that the network level two-fluid model is representative of the average behaviour of a driver. During the collection of data in these cities the chase car methodology was used to ensure this.

Using the data shown in Table 1, Eq. (21) was estimated using linear regression to estimate an average behaviour. Based on the regression shown in Fig. 2, the parameter estimates for w and β, were found to be 1.494 and 0.787 respectively. In the regression model shown in Fig. 2, it is assumed that w and β are constant, with regard to variations in network features.

This implies that the perceived probability of causing a crash is $\alpha f_r^{0.787}$. The parameter 'w' in conjunction with the two-fluid parameter 'n' could be used to estimate the utility of crashing. For example, drivers' in Mexico City were found to have the lowest perceived severity of a crash $-1.494 v_r^{1.61}$, while Arlington 2 had the highest perceived severity $-1.494 v_r^{2.64}$.

Though the regression model in Fig. 2 has a fairly reasonable fit of an R^2 of 0.64, the effect of network features in explaining the variance in the two-fluid model data was further investigated. The effects of geometric features and signals on the two-fluid parameters (T_m and n) were extensively studied by Ayadh (1986), Ardekani et al. (1992) and Bhat et al. (1992). Based on our new understanding regarding the relationship between T_m and n, and that the 'true' parameters that determine the two-fluid model are the perceived crash likelihood factor (β), perceived impact factor (k) and the crash weighting factor (w), there is a need to estimate the effects of network features on these parameters.

A simple linear regression was undertaken to determine network features that have a significant effect on perceived impact factor (k). The model was developed based on a backward elimination approach, till all the variables that were left in the model were significant with a p-value of less than 0.10. The results for the estimates for the perceived impact factor are shown in Table 2. Additional analysis on estimates for the perceived crash likelihood factor (β) and crash weighting factor (w) are shown in Table 3.

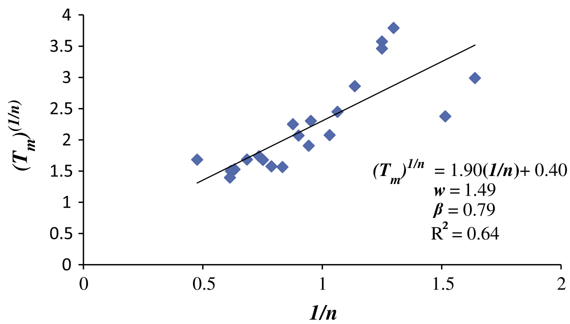

Fig. 2. Relationship between parameters of the two-fluid models 1980–1990.

Table 2
Estimates for the perceived impact factor (k).

K	Coefficient	Standard error	t-Value	$P > \|t\|$	95% Confidence interval	
X_2	−1.169	0.506	−2.31	0.035	−2.2426	−0.0956
X_3	0.147	0.078	1.87	0.080	−0.0194	0.3127
X_4	0.005	0.001	3.59	0.002	0.0022	0.0085
X_9	0.502	0.224	2.24	0.039	0.0275	0.9768
Constant	0.750	0.333	2.25	0.039	0.0431	1.4566

Table 3
Estimates for the perceived crash likelihood factor (β) and crash weighting factor (w), using the backward elimination approach.

	Coefficient	Standard error	t-Value	$P > \|t\|$	95% Confidence interval	
B						
X_2	−0.295	0.134	−2.20	0.027	−0.558	−0.033
Const	1.075	0.275	3.90	0.000	0.536	1.615
W						
Const	2.081	1.053	1.98	0.048	0. 180	4.144

$$k = 0.75 - 1.169X_2 + 0.147X_3 + 0.005X_4 + 0.502X_9 \quad R^2 = 0.58 \tag{22}$$

It was observed that the fraction of one way streets, the average number of lanes per street, signal density and the fraction of actuated signals had a significant effect on the perceived impact factor. As the perceived impact factor reduces, the driver's perception of loss caused due to a crash at a certain velocity reduces. Reduction in the number of lanes (X_3) reduces the possibility of lane changes and the need to accelerate and decelerate and ensures smooth flow of traffic, similarly reduction in intersection density (X_4) and increase in fraction of one way streets (X_2) also reduces the frequency of stops and results in a smooth flow of traffic. Reduction in the frequency of stops makes drivers feel safer, and therefore reduce their perception of loss caused due to a crash at a certain velocity, and therefore reduces the perceived impact factor. Increase in fraction of actuated signals (X_9) was also found to increase the perceived impact factor. Actuated signals results in higher average speeds in the network, therefore a driver if involved in a crash would have more severe consequences, than in a network with lower speeds. Tarko (2009) also found that the intersection density had a significant impact on the "*perceived risk*", which was essentially the perceived impact factor in the model discussed in this paper. The R-squared value was found to be low, but other geometric factors and factors related to driving culture that were not measured could improve the explanatory power.

Earlier, the perceived crash likelihood factor (β) and crash weighting factor (w) were estimated as constants (Fig. 2) by minimizing the error shown in the following equation:

$$error = \left[(T_m)^{\frac{1}{n}} - \frac{w}{n\beta} - w\left(\frac{1}{\beta} - 1\right)\right]^2 \tag{23}$$

In order to estimate the effects of network characteristics, β and w were assumed to be linear functions of the network features. Therefore, $\beta = BX$ and $w = WX$. Where, X is the vector of variables X_1–X_{10}, while B and W are coefficients to be estimated by minimizing as follows:

$$error = \left[(T_m)^{\frac{1}{n}} - \frac{(WX')}{n(BX')} - (WX')\left(\frac{1}{(BX' - 1)}\right)\right]^2 \tag{24}$$

The sum of the errors shown in Eq. (24) was minimized in STATA, to estimate B and W.

A backward elimination approach was used to determine the most reasonable model, which is shown in Table 3. Based on Table 3, $\beta = 1.075 - 0.295X_2$ and $w = 2.081$. Therefore, Eq. (21) becomes:

$$(T_m)^{\frac{1}{n}} = \frac{2.081}{1.075 - 0.295X_2}\left(\frac{n+1}{n}\right) - 2.081 \quad R^2 = 0.89 \tag{25}$$

The fraction of one-way streets (X_2) was found to have a significant effect on the perceived crash likelihood factor (β). It was found that as the fraction of one-way streets increased the perceived crash likelihood factor decreased, and therefore the perceived probability of causing a crash increases for a given ratio of average speed and running speed. One possible explanation for this could be that while a driver is driving in an environment with a larger fraction of one-way streets, stopping could result in a likelihood of rear end crashes by following vehicles. In order to compensate for this drivers reduce their perceived crash likelihood factor. No significant effects of network features were observed on the crash weighting factor. One possible reason for this could be other unobserved variables such as guard rails, divided vs. undivided roadway, and lane

Table 4
Estimates for the perceived crash likelihood factor (β) and crash weighting factor (w) with different variables.

Variable (Var)	B				W				R^2
	Coeff. var	p-Value	Coeff. const	p-Value	Coeff. var	p-Value	Coeff. const	p-Value	
X_1	0.001	0.050	0.416	0.200	.	.	1.700	0.100	0.851
X_2	−0.295	0.030	1.075	0.000	.	.	2.081	0.050	0.890
X_3	−0.013	0.820	0.812	0.020	.	.	1.444	0.160	0.640
X_4	−0.001	0.040	1.004	0.004	.	.	1.574	0.120	0.860
X_5	−0.001	0.050	0.743	0.090	.	.	1.109	0.290	0.870
X_6	0.008	0.380	0.646	0.030	.	.	1.779	10.000	0.670
X_7	0.002	0.450	0.682	0.030	.	.	1.660	0.110	0.670
X_8	0.121	0.510	0.620	0.190	.	.	1.144	0.320	0.660
X_9	0.205	0.050	0.997	0.000	.	.	2.537	0.030	0.840
X_{10}	−0.052	0.790	0.767	0.030	.	.	1.362	0.230	0.640
X_1	.	.	0.913	0.000	−0.004	0.190	3.594	0.100	0.910
X_2	.	.	0.982	0.000	1.274	0.140	1.721	0.030	0.930
X_3	.	.	0.782	0.020	0.068	0.710	1.267	0.230	0.660
X_4	.	.	0.862	0.000	0.003	0.190	1.060	0.120	0.900
X_5	.	.	0.749	0.050	0.003	0.250	1.097	0.220	0.930
X_6	.	.	0.872	0.000	−0.042	0.510	3.032	0.270	0.680
X_7	.	.	0.868	0.000	−0.009	0.490	2.516	0.200	0.700
X_8	.	.	0.694	0.090	−0.343	0.480	1.347	0.240	0.690
X_9	.	.	1.067	0.000	−0.955	0.160	2.936	0.020	0.890
X_{10}	.	.	0.746	0.050	0.160	0.760	1.295	0.270	0.650

striping that could help explain the perceived crash weighting factor, which were not captured in the data collection. More controlled experiments are required to understand factors affecting w. It is interesting to note that with the inclusion of geometric factors to explain the perceived crash likelihood factor the R-squared fit improved significantly from 0.64 to 0.89. This suggests that the coefficients of the geometric factors are able to capture variations in how drivers' perceive them in relationship to their likelihood of causing a crash.

To further ensure that the model presented in Table 3 and Eq. (24) is reasonable each of the ten variables X_1–X_{10} in the regression to explain β and w. The results are presented in Table 4. It is observed that in the model for w none of the coefficients for each of the variables were statistically significant, while models for β that included each of the variables X_1, X_2, X_4, X_5 and X_9 separately had coefficients that were statistically significant. Interestingly, consistent with the model developed using the backward elimination approach, the model containing X_2 in the equation for β had the highest R^2 value with a statistically significant coefficient for a variable, and therefore Eq. (24) is considered the most reasonable model.

An interesting finding in Table 4 is that apart from X_2, coefficients for X_1, X_4, X_5 and X_9 are statistically significant. The finding that the coefficient of X_1 and X_9 are positive, suggests that as the as the block length (X_1) or fraction of actuated signals (X_9) increases the perceived crash likelihood factor increases. However, the negative coefficients for X_4 and X_5, suggests that as the as the intersection density (X_4) or signal density (X_5) increases, the perceived crash likelihood factor decreases. Therefore, an increase in X_1 and X_9, and a decrease in X_4 and X_5 results in a decrease in the perceived probability of causing a crash for a given ratio of average speed and running speed.

Though speed fluctuations and other characteristics were not structurally included in the perceived probability to cause a crash or in the utility model, their effects can be analyzed by studying their impacts on β and k through regression analysis. Unfortunately since the data was not available, the effects on the parameters due to factors known to affect crashes, such as: speed fluctuation, number of vehicle stops, queue lengths, percentage of left turns, speed distribution and deceleration distribution (FHWA, 2008), could not be evaluated.

The findings above were based on data collected in 1980s and 1990s.[2] It is possible that with additional safety technologies, people might behave differently, as demonstrated by Evans and Graham (1991) on the effect of seat belts on risk compensation in driving behaviour. It should be expected that as technologies improve effects on perceived impact factor (k), perceived crash likelihood factor (β) and crash weighting factor (w) should be expected. But an important question, regarding the stability of the underlying theory still remains. To further test robustness of the predictions of the model with respect to the relationship between T_m and n, data from a more recent study collected during August 2002–July 2003 by Jones and Farhat (2004) on arterial streets was analyzed. They showed the existence of the two-fluid model on arterial streets. To further test the predictions of the theory (Eq. (21)). Eq. (21) was regressed and the high R^2 value observed in Fig. 3. Further, using the model shown in Table 3, β was estimated. The estimated β was found to be greater than k. This provides further evidence for the theory proposed in this paper.

Utilizing the two-fluid model data collected for various cities in the United States, tested the theoretical predictions of the behavioural framework for the two-fluid model. A method is also described to estimate the effects of network characteristics

[2] A dummy variable for all data points collected in 1990s. It was not found to have significant effects on any of the above model parameters k, β or w.

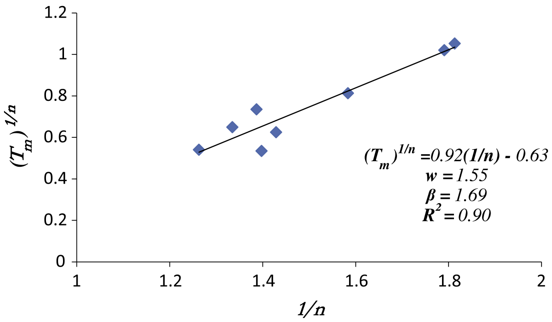

Fig. 3. Relationship between the two-fluid model parameters for Jones and Farhat (2004).

on the driver's perception, specifically on the perceived crash likelihood factor (β), perceived impact factor (k) and the crash weighting factor (w).

The main contribution of the paper is that the two-fluid model network data collected by Bhat et al. (1992) and Jones and Farhat (2004) provide reasonable evidence for the underlying relationship between T_m and n. This underlying relationship was not discovered earlier, and was predicted by the theory proposed in this paper. Though the sample size was limited, additional analysis was undertaken to explore which network features have a significant effect on the perceived impact factor (k), crash likelihood factor (β) and crash weighting factor (w).

5. Conclusion

The two-fluid model was developed based on an analogous system in particle physics. This paper presents a behavioural framework to understand the two-fluid model. The two-fluid model was derived using a state-dependent expected utility approach. Three parameters were found to affect the characteristics of the two-fluid model perceived crash likelihood factor (β), perceived impact factor (k) and the crash weighting factor (w). The crash likelihood factor determines how the driver perceives the probability of causing a crash, and was shown to be related to the power function studied by Elvik et al. (2004). The perceived impact factor describes the driver's perception of the severity of a crash at a given speed, while the crash weighting factor determines how the driver's weigh the disutility of a crash. These parameters were found to describe the traditional parameters of the two-fluid model T_m and n. T_m was also found to be a decreasing function of n.

Using the two-fluid model data for various cities and arterial roads the functional relationship derived between T_m and n was tested. The data provided sufficient evidence for the predictions of the theory proposed in this study, which predicted an underlying relationship between T_m and n. Further studies need to be undertaken to provide more evidence for this phenomenon. A method was also presented to determine the effects of network characteristics on the perceived crash likelihood factor, perceived impact factor and the crash weighting factor. It was found that fraction of one way streets, average number of lanes per street, signal density and fraction of actuated signals had a significant effect on the perceived impact factor (k). Also, increase in fraction of one-way streets was found to decrease the perceived crash likelihood factor (β), no network characteristics were observed to influence the crash weighting factor (w). There is a need to conduct further research to determine impact of network characteristics and other traffic characteristics such as speed fluctuations number of stops on these parameters.

This paper provides a framework to use the two-fluid model to evaluate the driver's perception of causing a crash. For instance, if a driver has a lower perceived crash likelihood factor as compared to the average population, it can be concluded that this driver considers him/herself more likely to be involved in a crash. In the case of newly licensed drivers it is possible that the perceived crash likelihood factor is skewed and could be on the higher side. Training programs and education can be undertaken to promote the convergence of the perceived crash likelihood factor to acceptable levels. Higher than average values of perceived impact factor suggest that a driver considers him/her self more likely to be involved in a more severe accident. Aggressive and over confident drivers are expected to have lower values of perceived impact factor. This could be used to develop insurance and penalty mechanisms that would promote safer driving behaviour. More research is required to better understand and utilize these parameters to promote engineering solutions directed towards behaviour.

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